Fifth SI Edition

CHAPTER

6

MECHANICS OF MATERIALS

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Shearing Stresses in Beams and Thin-Walled Members

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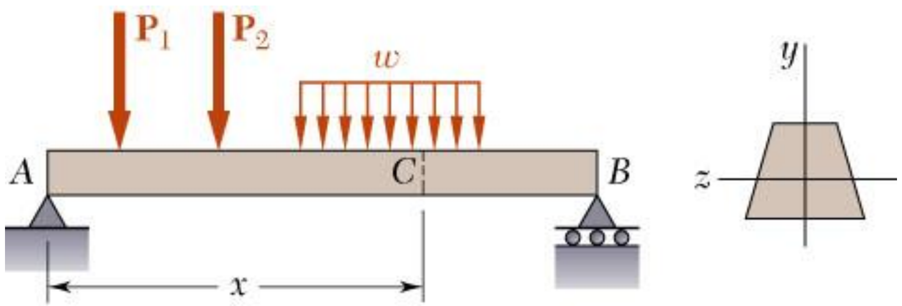
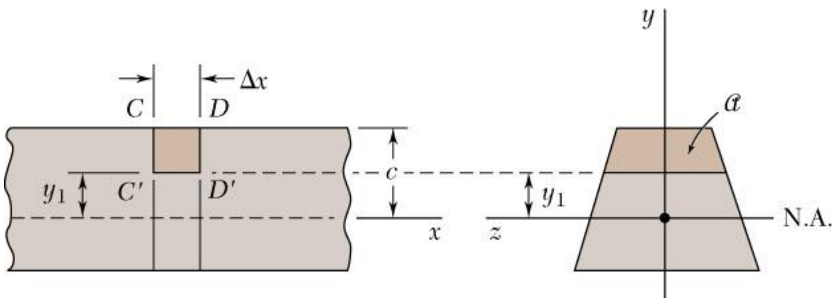
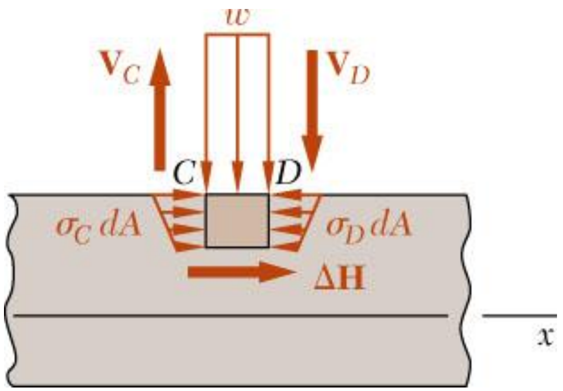
Shear on the Horizontal Face of a Beam Element







- Consider prismatic beam
- For equilibrium of beam element

$$\sum F_x = 0 = \Delta H + \int_A (\sigma_D - \sigma_C)\, dA$$

$$\Delta H = \frac{M_D - M_C}{I} \int_A y\, dA$$

- Note,

$$Q = \int_A y\, dA$$

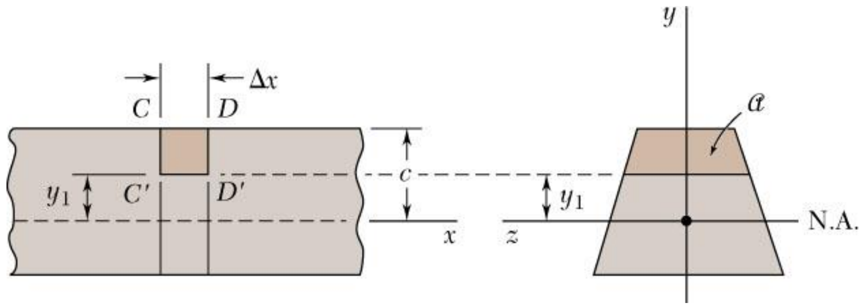
$$M_D - M_C = \frac{dM}{dx} \Delta x = V\, \Delta x$$

- Substituting,

$$\Delta H = \frac{VQ}{I} \Delta x$$

$$q = \frac{\Delta H}{\Delta x} = \frac{VQ}{I} = shear\ \ flow$$

Shear on the Horizontal Face of a Beam Element



- Shear flow,

$$q = \frac{\Delta H}{\Delta x} = \frac{VQ}{I} = shear\ \ flow$$

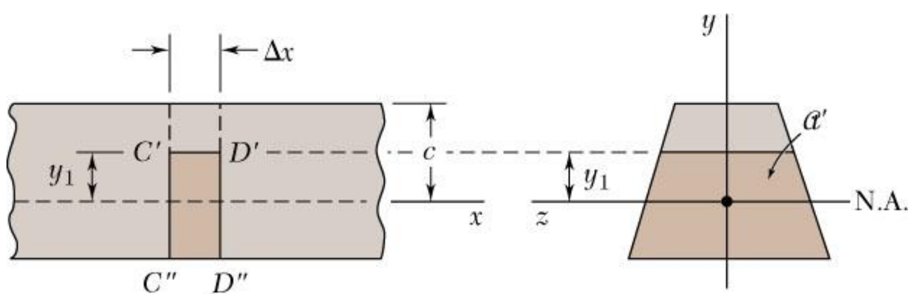
- where

$$Q = \int_A y\,dA$$

$$= \text{first moment of area above } y_1$$

$$I = \int_{A+A'} y^2 dA$$

$$= \text{second moment of full cross section}$$



- Same result found for lower area

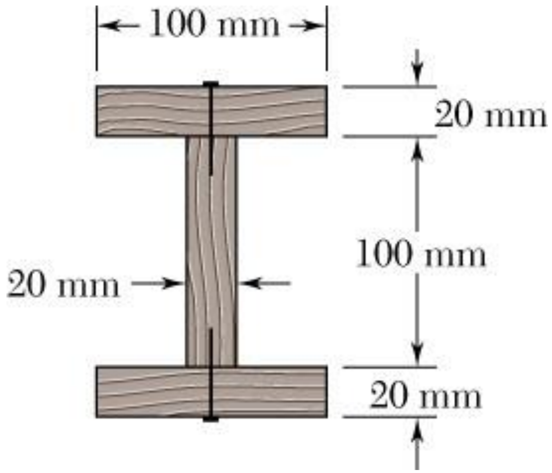
$$q' = \frac{\Delta H'}{\Delta x} = \frac{VQ'}{I} = -q'$$

$$Q + Q' = 0$$

$$= \text{first moment with respect to neutral axis}$$

$$\Delta H' = -\Delta H$$

Concept Application 6.1

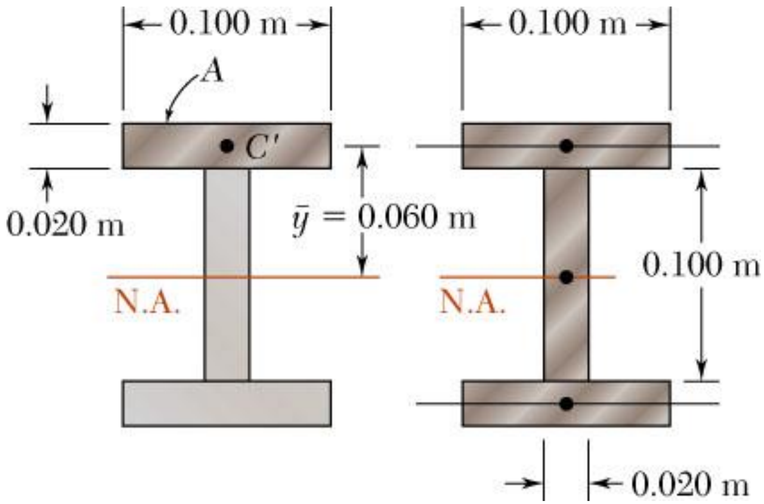


A beam is made of three planks, nailed together. Knowing that the spacing between nails is 25 mm and that the vertical shear in the beam is V = 500 N, determine the shear force in each nail.

SOLUTION:

- Determine the horizontal force per unit length or shear flow q on the lower surface of the upper plank.
- Calculate the corresponding shear force in each nail.

Concept Application 6.1



$$Q = A\bar{y}$$
$$= (0.020\,\text{m} \times 0.100\,\text{m})(0.060\,\text{m})$$
$$= 120 \times 10^{-6}\,\text{m}^3$$
$$I = \tfrac{1}{12}(0.020\,\text{m})(0.100\,\text{m})^3$$
$$+ 2[\tfrac{1}{12}(0.100\,\text{m})(0.020\,\text{m})^3$$
$$+ (0.020\,\text{m} \times 0.100\,\text{m})(0.060\,\text{m})^2]$$
$$= 16.20 \times 10^{-6}\,\text{m}^4$$

SOLUTION:

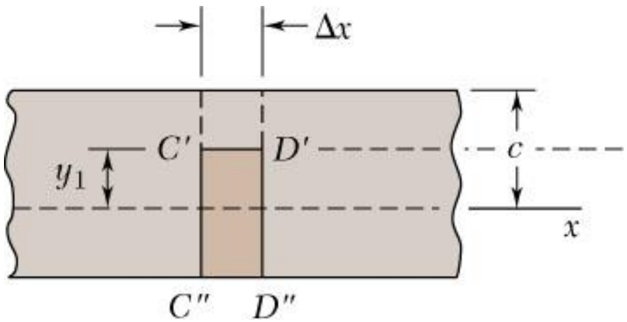
- Determine the horizontal force per unit length or shear flow q on the lower surface of the upper plank.

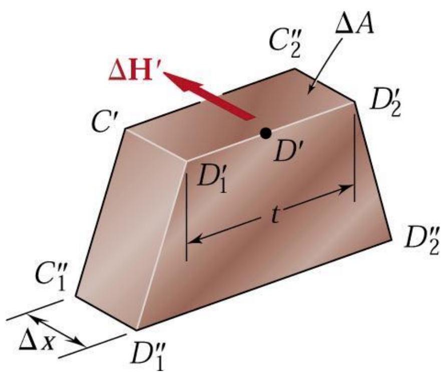
$$q = \frac{VQ}{I} = \frac{(500\,\text{N})(120 \times 10^{-6}\,\text{m}^3)}{16.20 \times 10^{-6}\,\text{m}^4}$$
$$= 3704\,\text{N/m}$$

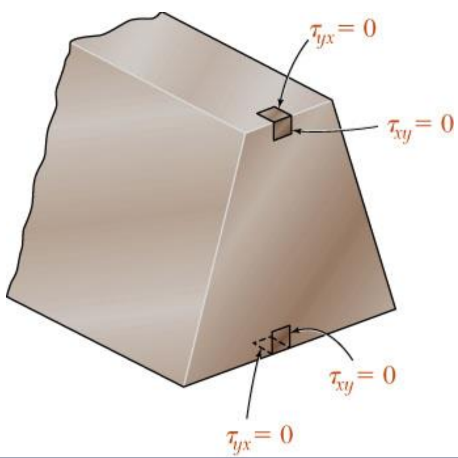
- Calculate the corresponding shear force in each nail for a nail spacing of 25 mm.

$$F = (0.025\,\text{m})q = (0.025\,\text{m})(3704\,N/m$$
$$\boxed{F = 92.6\,\text{N}}$$

Determination of the Shearing Stress in a Beam







- The *average* shearing stress on the horizontal face of the element is obtained by dividing the shearing force on the element by the area of the face.

$$\tau_{ave} = \frac{\Delta H}{\Delta A} = \frac{q\,\Delta x}{\Delta A} = \frac{VQ}{I}\frac{\Delta x}{t\,\Delta x}$$

$$= \frac{VQ}{It}$$

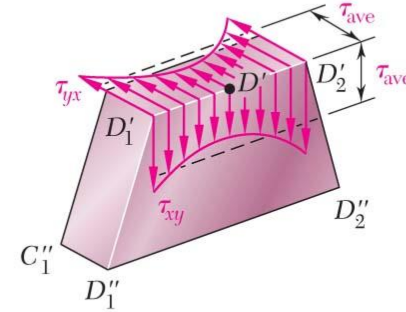


Fig. 6.12

- On the upper and lower surfaces of the beam, τ_{yx}= 0. It follows that τ_{xy}= 0 on the upper and lower edges of the transverse sections.

- *b<=h/4,C1及C2點之剪應力值*

  不會超過沿中性軸之應力

  平均值0.8% (p377)

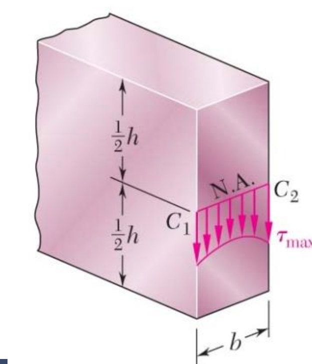


Fig. 6.14

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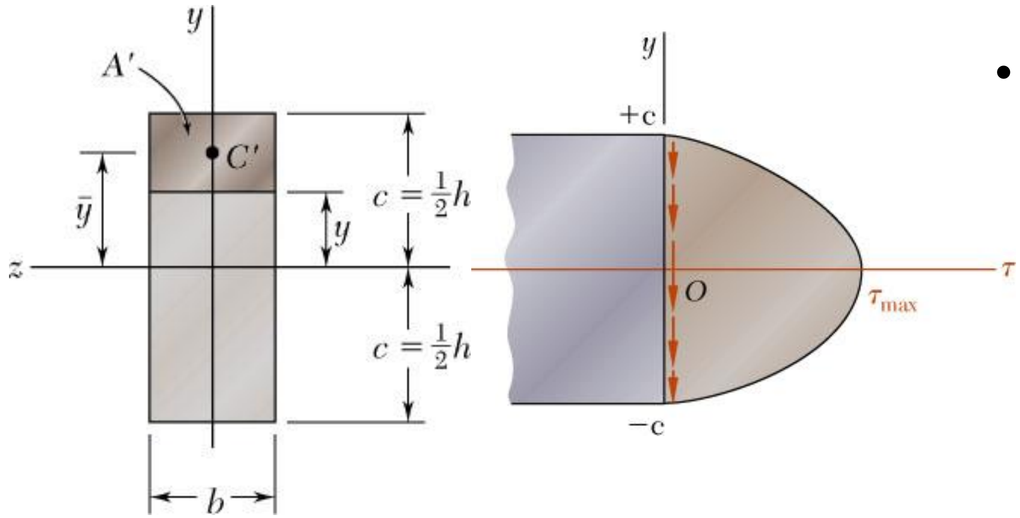
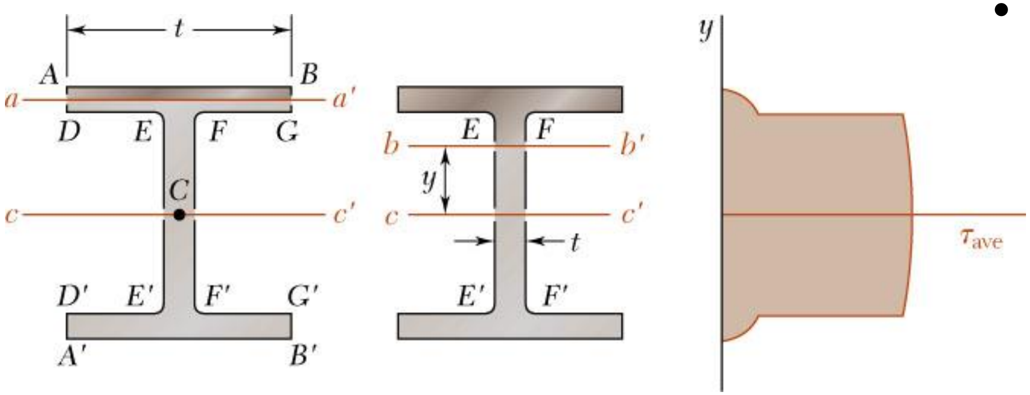
Shearing Stresses τ_{xy} in Common Types of Beams



- For a narrow rectangular beam,

$$\tau_{xy} = \frac{VQ}{Ib} = \frac{3}{2}\frac{V}{A}\left(1 - \frac{y^2}{c^2}\right)$$

$$\tau_{max} = \frac{3}{2}\frac{V}{A}$$



- For American Standard (S-beam) and wide-flange (W-beam) beams

$$\tau_{ave} = \frac{VQ}{It}$$

$$\tau_{max} = \frac{V}{A_{web}}$$

Concept Application 6.2

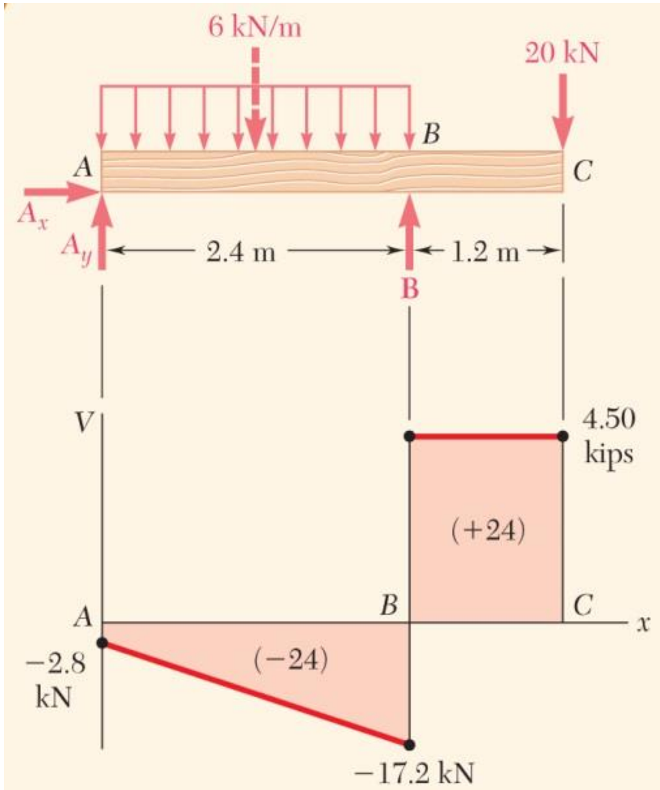
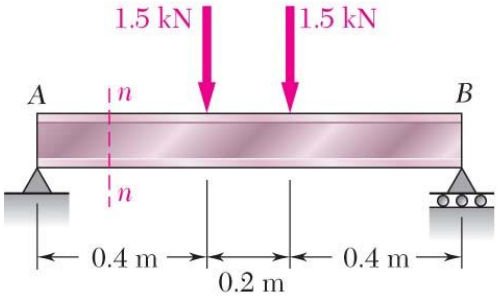
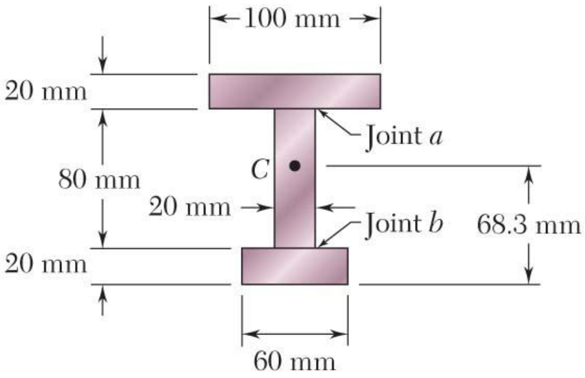


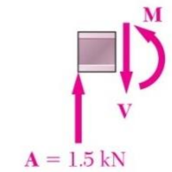
Fig. 5.19 *(repeated)*

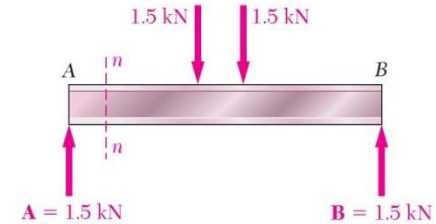
τ_{all}=1.75MPa, Check that the design is acceptable from the point of view of the shear stresses

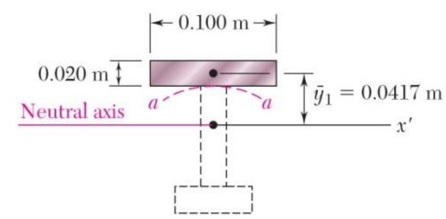
Sample Problem 6.1

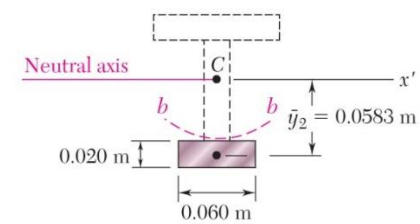




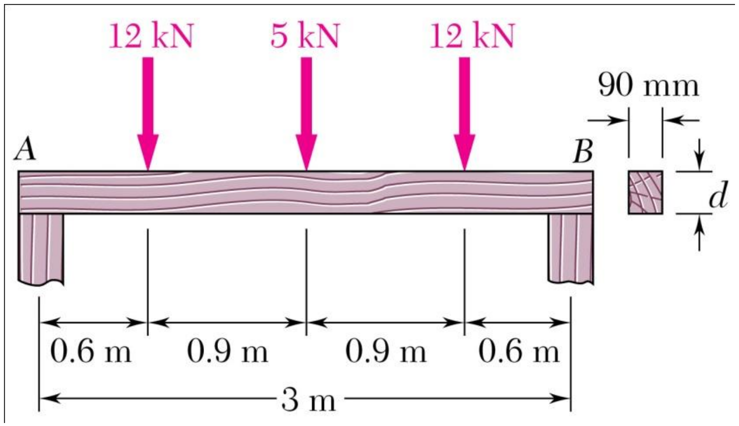








Sample Problem 6.2



A timber beam is to support the three concentrated loads shown. Knowing that for the grade of timber used,

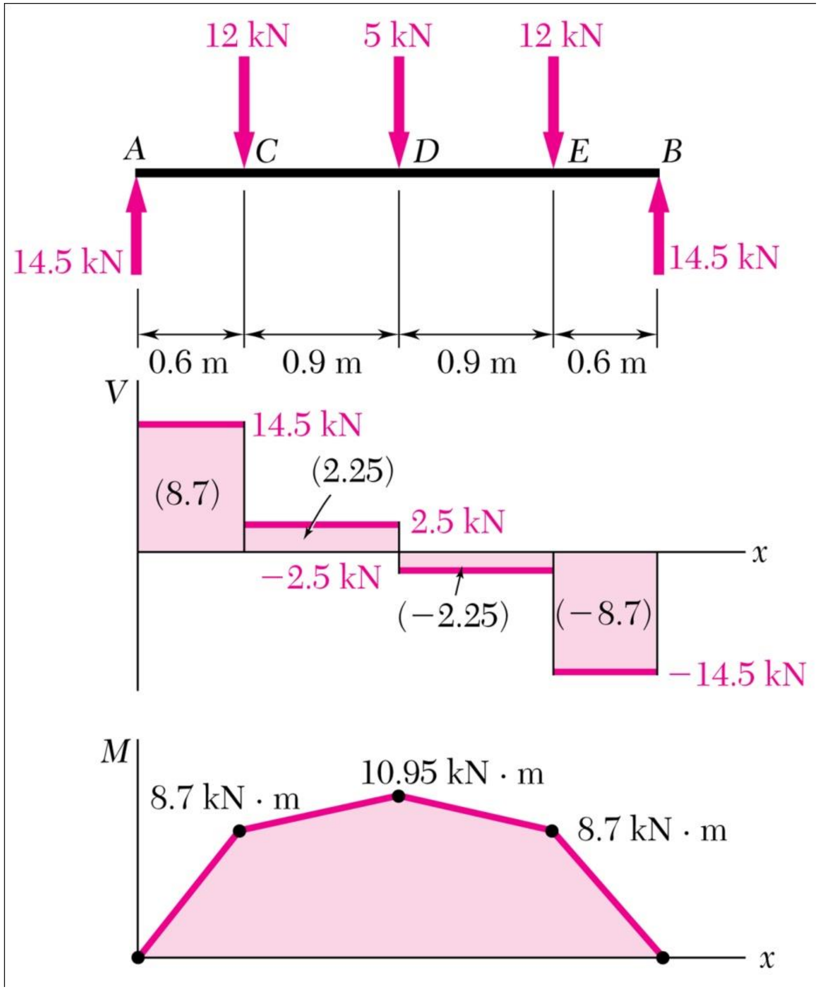
$$\sigma_{all} = 12\,\text{MPa} \qquad \tau_{all} = 0.8\,\text{MPa}$$

determine the minimum required depth *d* of the beam.

SOLUTION:

- Develop shear and bending moment diagrams. Identify the maximums.
- Determine the beam depth based on allowable normal stress.
- Determine the beam depth based on allowable shear stress.
- Required beam depth is equal to the larger of the two depths found.

Sample Problem 6.2



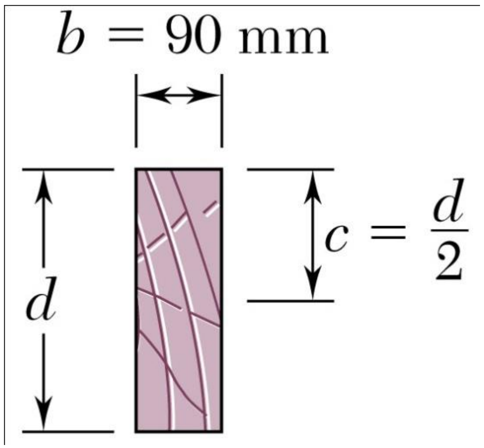
SOLUTION:

Develop shear and bending moment diagrams. Identify the maximums.

$$V_{max} = 14.5\,\text{kN}$$

$$M_{max} = 10.95\,\text{kNm}$$

Sample Problem 6.2



$$I = \tfrac{1}{12} b d^3$$

$$S = \frac{I}{c} = \tfrac{1}{6} b d^2$$

$$= \tfrac{1}{6}(0.09\,\text{m}) d^2$$

$$= (0.015\,\text{m}) d^2$$

- Determine the beam depth based on allowable normal stress.

$$\sigma_{all} = \frac{M_{\max}}{S}$$

$$12 \times 10^6\ \text{Pa} = \frac{10.95 \times 10^3\ \text{Nm}}{(0.015\,\text{m}) d^2}$$

$$d = 0.246\,\text{m} = 246\text{mm}$$

- Determine the beam depth based on allowable shear stress.

$$\tau_{all} = \frac{3}{2} \frac{V_{\max}}{A}$$

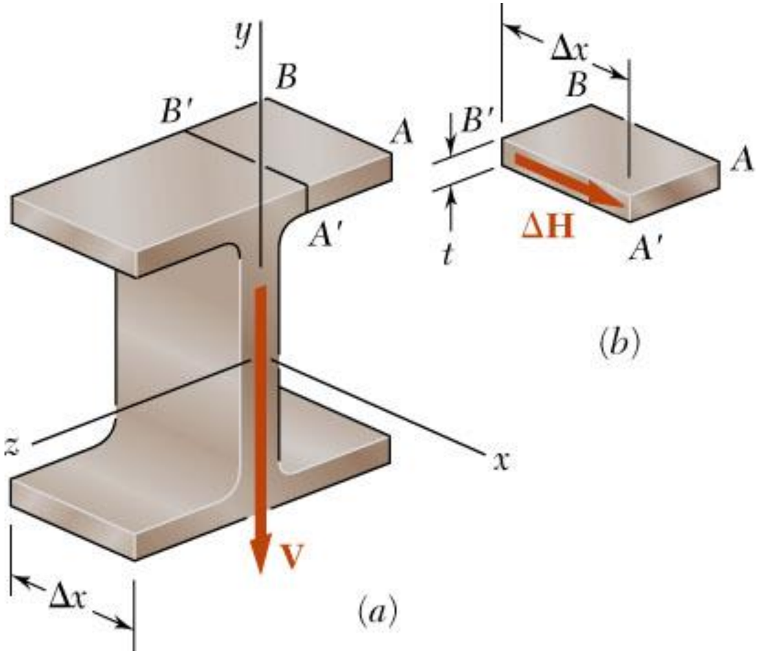
$$0.8 \times 10^6\ \text{Pa} = \frac{3}{2} \frac{14500}{(0.09\,\text{m}) d}$$

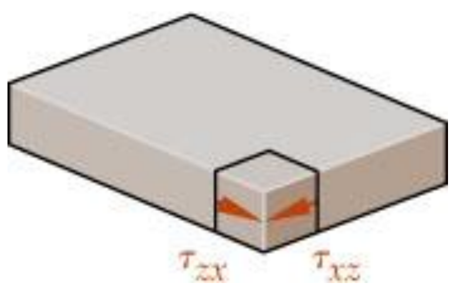
$$d = 0.322\,\text{m} = 322\,\text{mm}$$

- Required beam depth is equal to the larger of the two.

$$\boxed{d = 322\ \text{mm}}$$

Shearing Stresses in Thin-Walled Members





- Consider a segment of a wide-flange beam subjected to the vertical shear V.
- The longitudinal shear force on the element is

$$\Delta H = \frac{VQ}{I}\Delta x$$

- The corresponding shear stress is

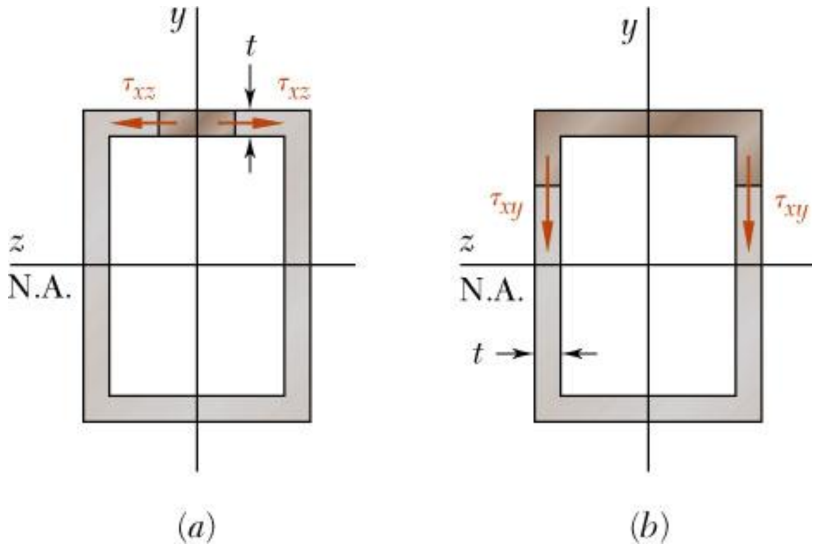
$$\tau_{zx} = \tau_{xz} \approx \frac{\Delta H}{t\,\Delta x} = \frac{VQ}{It}$$

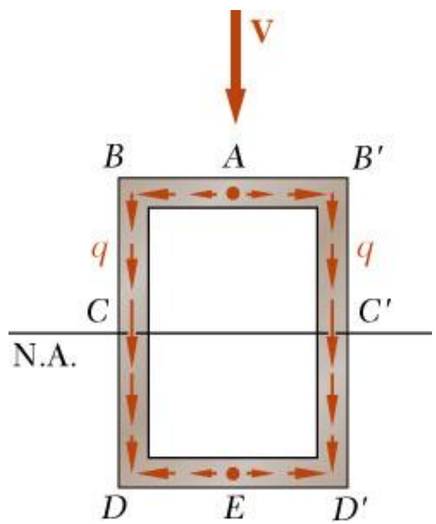
- Previously found a similar expression for the shearing stress in the web

$$\tau_{xy} = \frac{VQ}{It}$$

- NOTE: $\tau_{xy} \approx 0$ in the flanges
  $\tau_{xz} \approx 0$ in the web

Shearing Stresses in Thin-Walled Members





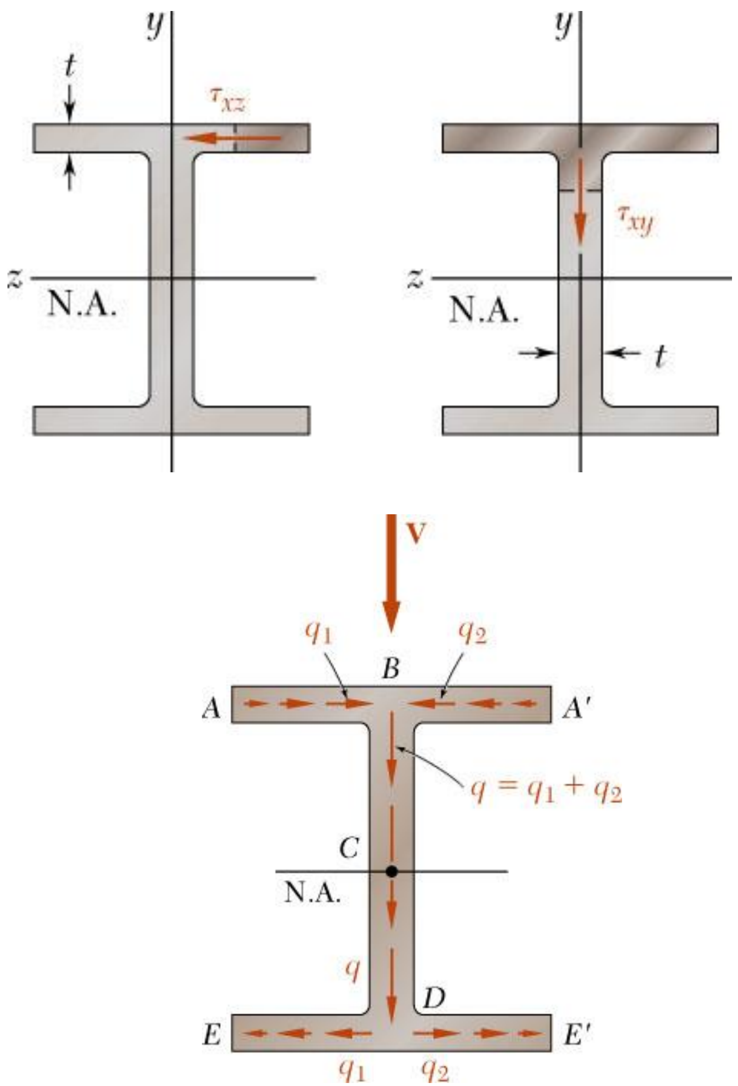
- The variation of shear flow across the section depends only on the variation of the first moment.

$$q = \tau t = \frac{VQ}{I}$$

- For a box beam, *q* grows smoothly from zero at A to a maximum at *C* and *C'* and then decreases back to zero at *E*.

- The sense of *q* in the horizontal portions of the section may be deduced from the sense in the vertical portions or the sense of the shear *V*.

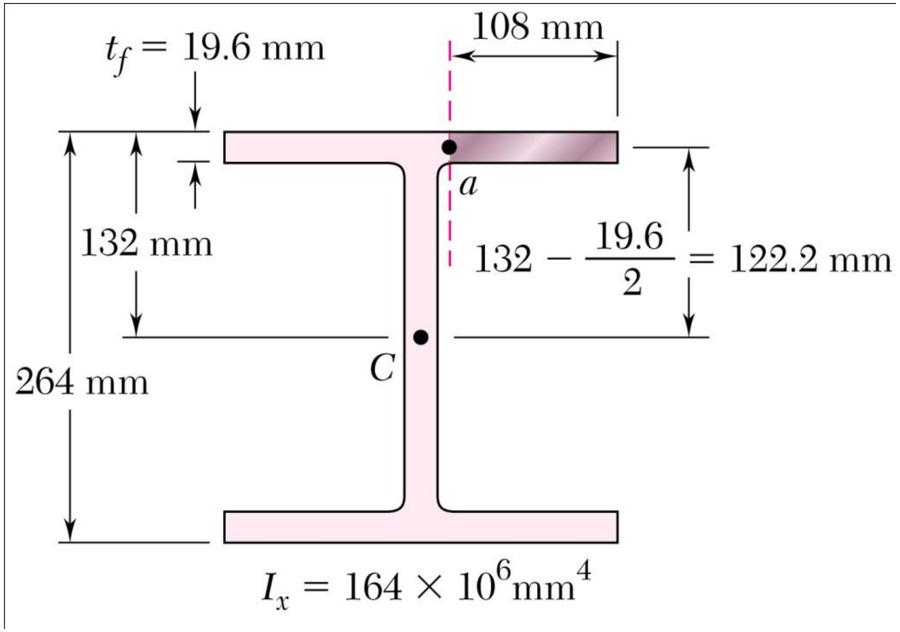
Shearing Stresses in Thin-Walled Members



- For a wide-flange beam, the shear flow increases symmetrically from zero at *A* and *A'*, reaches a maximum at *C* and then decreases to zero at *E* and *E'*.

- The continuity of the variation in *q* and the merging of *q* from section branches suggests an analogy to fluid flow.

Sample Problem 6.3



Knowing that the vertical shear is 200 kN in a W250x101 rolled-steel beam, determine the horizontal shearing stress in the top flange at the point *a*.

SOLUTION:

- For the shaded area,

$$Q = (108\,\text{mm})(19.6\,\text{mm})(122.2\,\text{mm})$$
$$= 258700\,\text{mm}^3$$

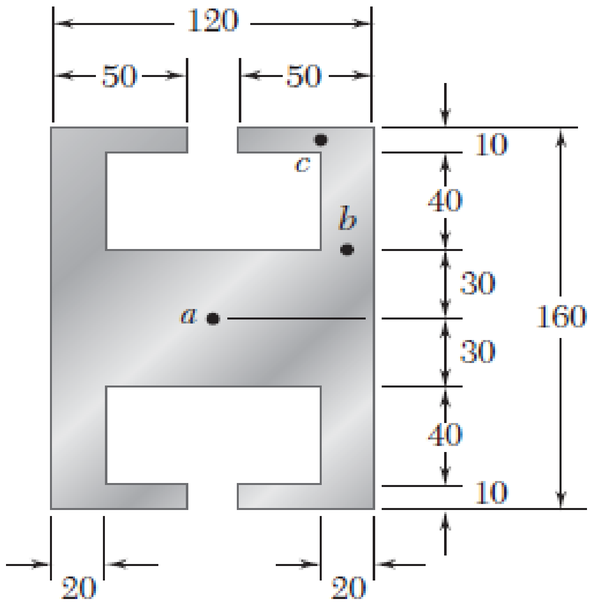
- The shear stress at *a*,

$$\tau = \frac{VQ}{It} = \frac{(200\times10^3\,\text{N})(258.7\times10^{-6}\,\text{m}^3)}{(164\times10^{-6}\,\text{m}^4)(0.0196\,\text{m})}$$

$$\boxed{\tau = 16.1\,\text{MPa}}$$

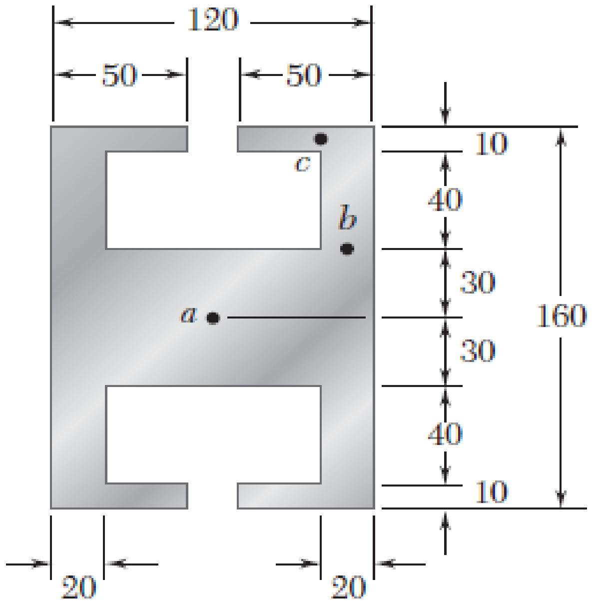
Problems 6.37

- Knowing that a given vertical shear **V** causes a maximum shearing stress of 75 MPa in an extruded beam having the cross section shown, determine the shearing stress at the three points indicated.



Dimensions in mm

Problems 6.37



Dimensions in mm

$$\tau = \frac{VQ}{It} \qquad \tau \text{ is proportional to } Q/t.$$

Point c:

$$Q_c = (30)(10)(75) = 22.5 \times 10^3\,\text{mm}^3$$

$$t_c = 10\text{ mm}$$

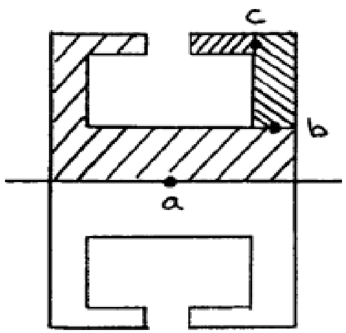
$$Q_c/t_c = 2250\text{ mm}^2$$

Point b:

$$Q_b = Q_c + (20)(50)(55) = 77.5 \times 10^3\,\text{mm}^3$$

$$t_b = 20\text{ mm}$$

$$Q_b/t_b = 3875\text{ mm}^2$$



Point a:

$$Q_a = 2Q_b + (120)(30)(15) = 209 \times 10^3\,\text{mm}^3$$

$$t_a = 120\text{ mm}$$

$$Q_a/t_a = 1741.67\text{ mm}^2$$

$(Q/t)_m$ occurs at b. $\quad \tau_m = \tau_b = 75\text{ MPa}$

$$\frac{\tau_a}{Q_a/t_a} = \frac{\tau_b}{Q_b/t_b} = \frac{\tau_c}{Q_c/t_c}$$

$$\frac{\tau_a}{1741.67\text{ mm}^2} = \frac{75\text{ MPa}}{3875\text{ mm}^2} = \frac{\tau_a}{2250\text{ mm}^2}$$

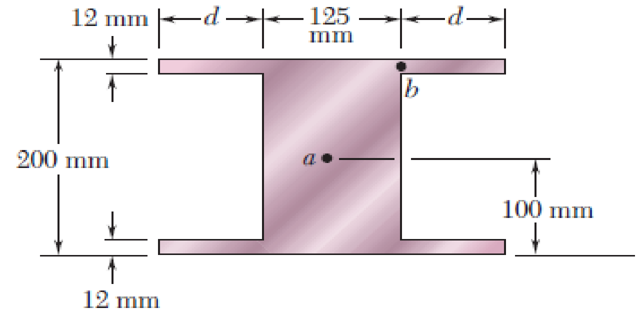
$\tau_a = 33.7\text{ MPa}$ ◀

$\tau_b = 75.0\text{ MPa}$ ◀

$\tau_c = 43.5\text{ MPa}$ ◀

Problems 6.38

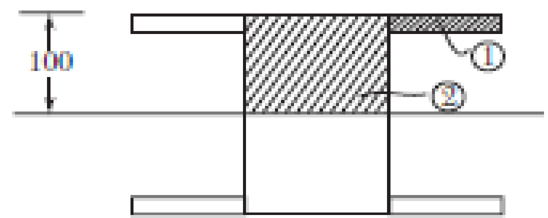
6.38 The vertical shear is 5.3 kN in a beam having the cross section shown. Knowing that $d = 100$ mm, determine the shearing stress (a) at point a, (b) at point b.



$$I_1 = \frac{1}{12}(100)(12)^3 + (100)(12)(94)^2 = 10.62 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{3}(125)(100)^3 = 41.67 \times 10^6 \text{ mm}^4$$

$$I = 4I_1 + 2I_2 = 125.82 \times 10^6 \text{ mm}^4$$



(a) $Q_a = 2A_1\bar{y}_1 + A_2\bar{y}_3$

$= (2)(100)(12)(94) + (125)(100)(50) = 850600 \text{ mm}^3$

$t_a = 125$ mm

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(5.3 \times 10^3)(850600)}{(125.82 \times 10^6)(125)} = 0.287 \text{ N/mm}^2 \text{ (i.e.) } 287 \text{ kPa}$$ ◄

$t_b = 12$ mm

(b) $Q_b = A_1\bar{y}_1 = (100)(12)(94) = 112800$

$$\tau_b = \frac{VQ_b}{It_b} = \frac{(5.3 \times 10^3)(112800)}{(125.82 \times 10^6)(12)} = 0.396 \text{ N/mm}^2 \quad \text{(i.e.) } 396 \text{ kPa}$$ ◄