Fifth SI Edition

MECHANICS OF MATERIALS

CHAPTER

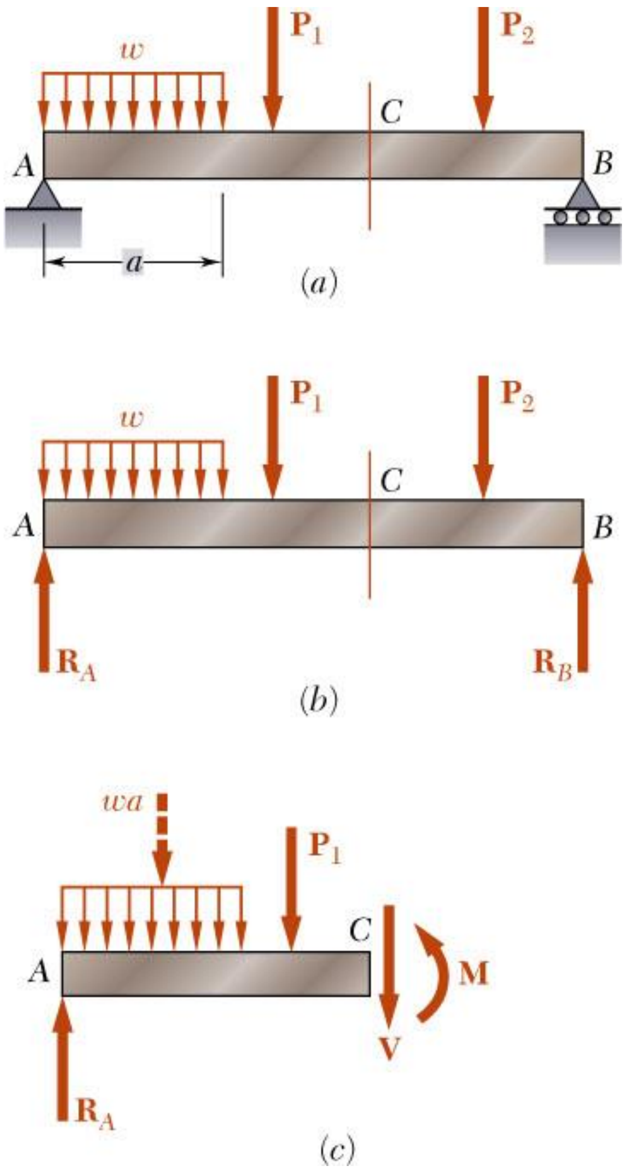
5

Ferdinand P. Beer
E. Russell Johnston, Jr.
John T. DeWolf
David F. Mazurek

Lecture Notes:
J. Walt Oler
Texas Tech University

Analysis and Design of Beams for Bending

Introduction



- Objective - Analysis and design of beams
- *Beams* - structural members supporting loads at various points along the member
- Transverse loadings of beams are classified as *concentrated* loads or *distributed* loads
- Applied loads result in internal forces consisting of a shear force (from the shear stress distribution) and a bending couple (from the normal stress distribution)
- Normal stress is often the critical design criteria

$$\sigma_x = -\frac{My}{I} \qquad \sigma_m = \frac{|M|c}{I} = \frac{|M|}{S}$$

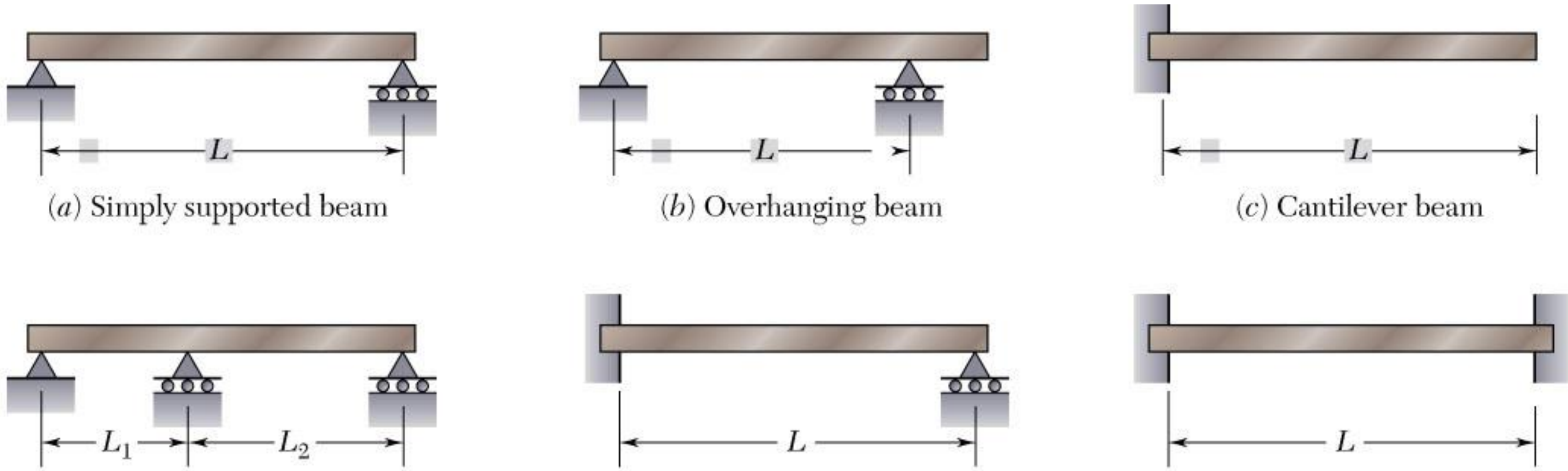
Requires determination of the location and magnitude of largest bending moment

Introduction

Classification of Beam Supports

Statically Determinate Beams

Statically Indeterminate Beams



(*a*) Simply supported beam

(*b*) Overhanging beam

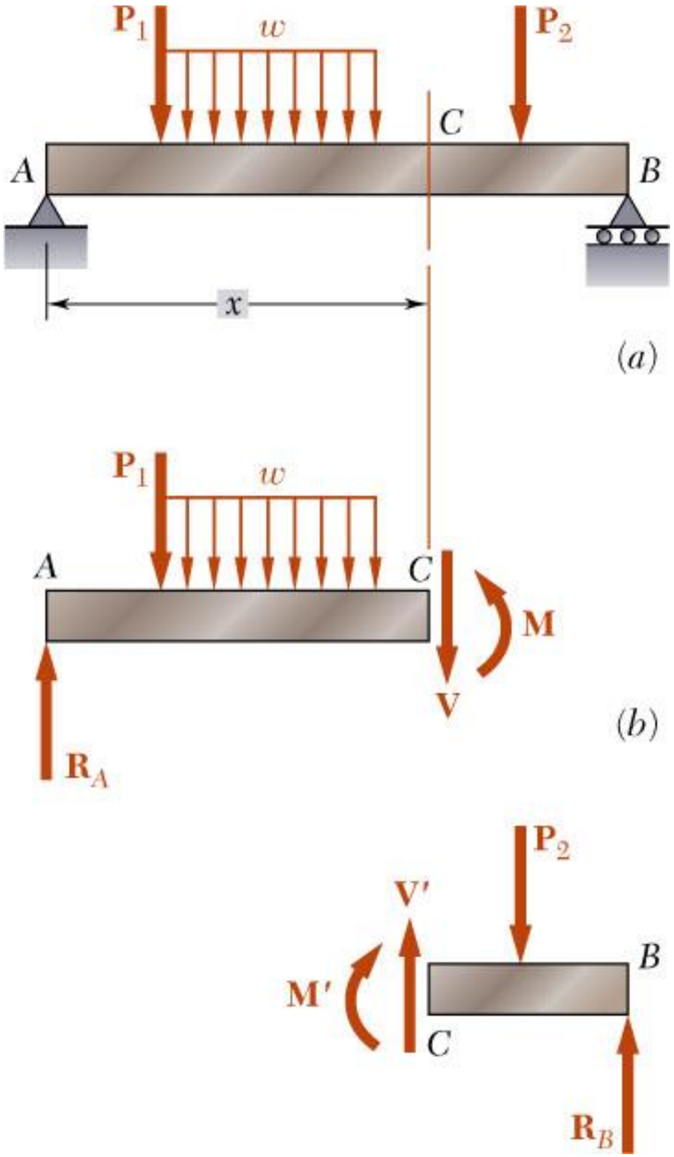
(*c*) Cantilever beam

(*d*) Continuous beam

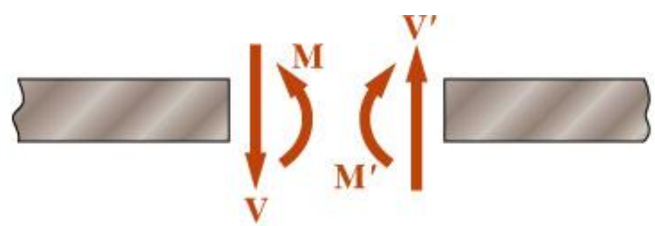
(*e*) Beam fixed at one end and simply supported at the other end

(*f*) Fixed beam

5.1 Shear and Bending Moment Diagrams p.348



- Determination of maximum normal and shearing stresses requires identification of maximum internal shear force and bending couple.
- Shear force and bending couple at a point are determined by passing a section through the beam and applying an equilibrium analysis on the beam portions on either side of the section.
- Sign conventions for shear forces V and V' and bending couples M and M'



(*a*) Internal forces
(positive shear and positive bending moment)

Concept Application 5.1

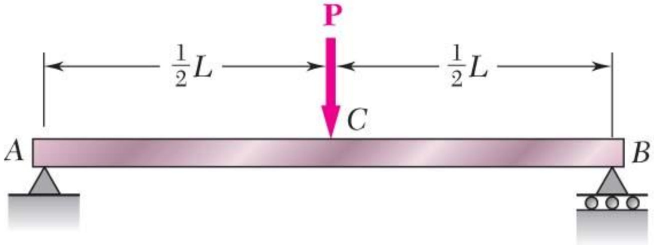


Fig. 5.8

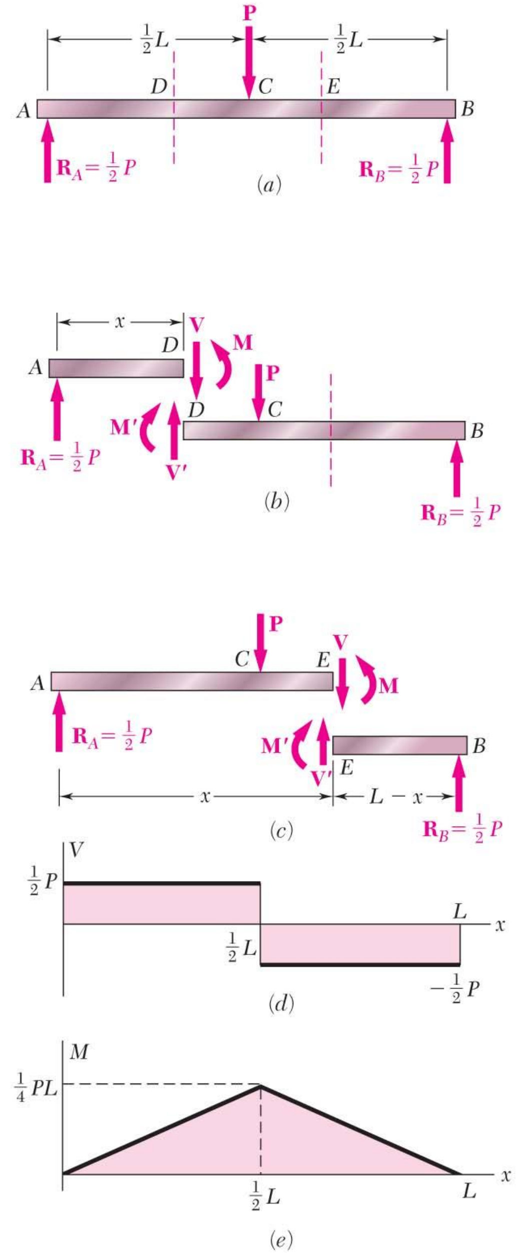
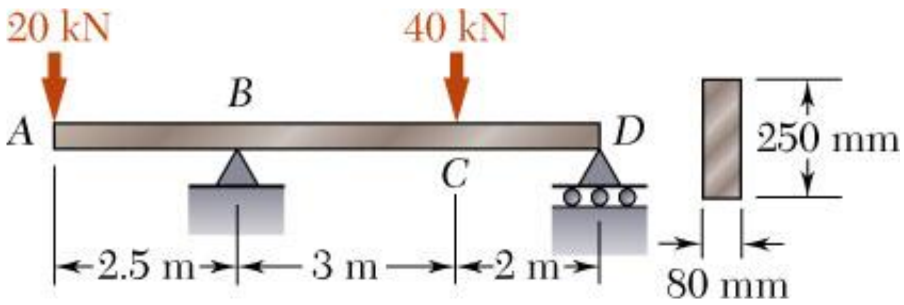


Fig. 5.9

Sample Problem 5.1

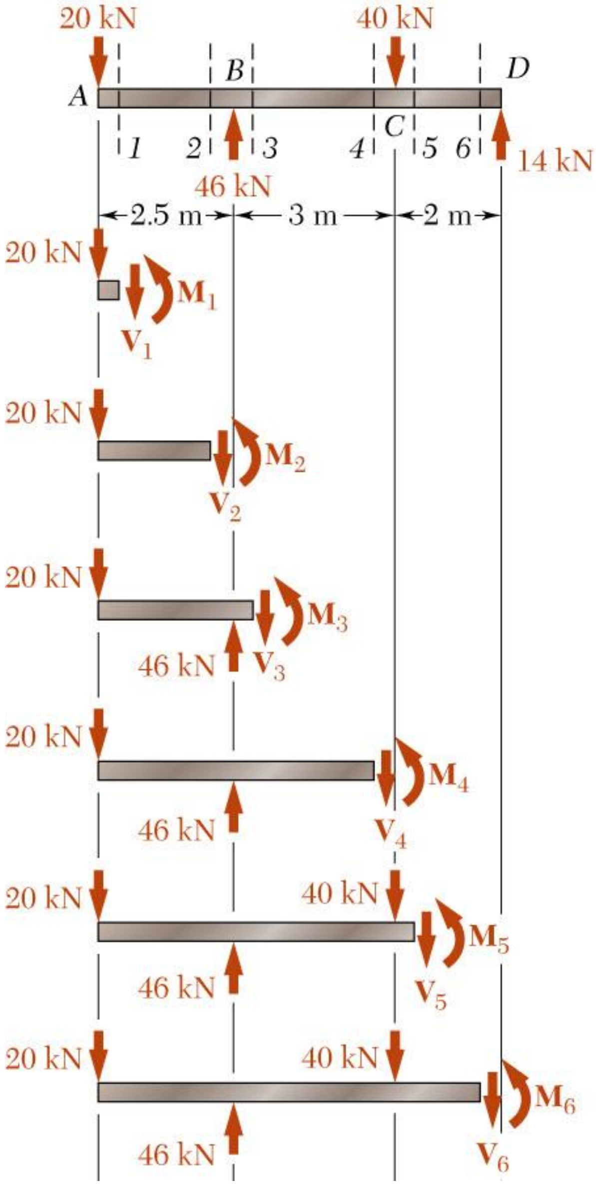


For the timber beam and loading shown, draw the shear and bend-moment diagrams and determine the maximum normal stress due to bending.

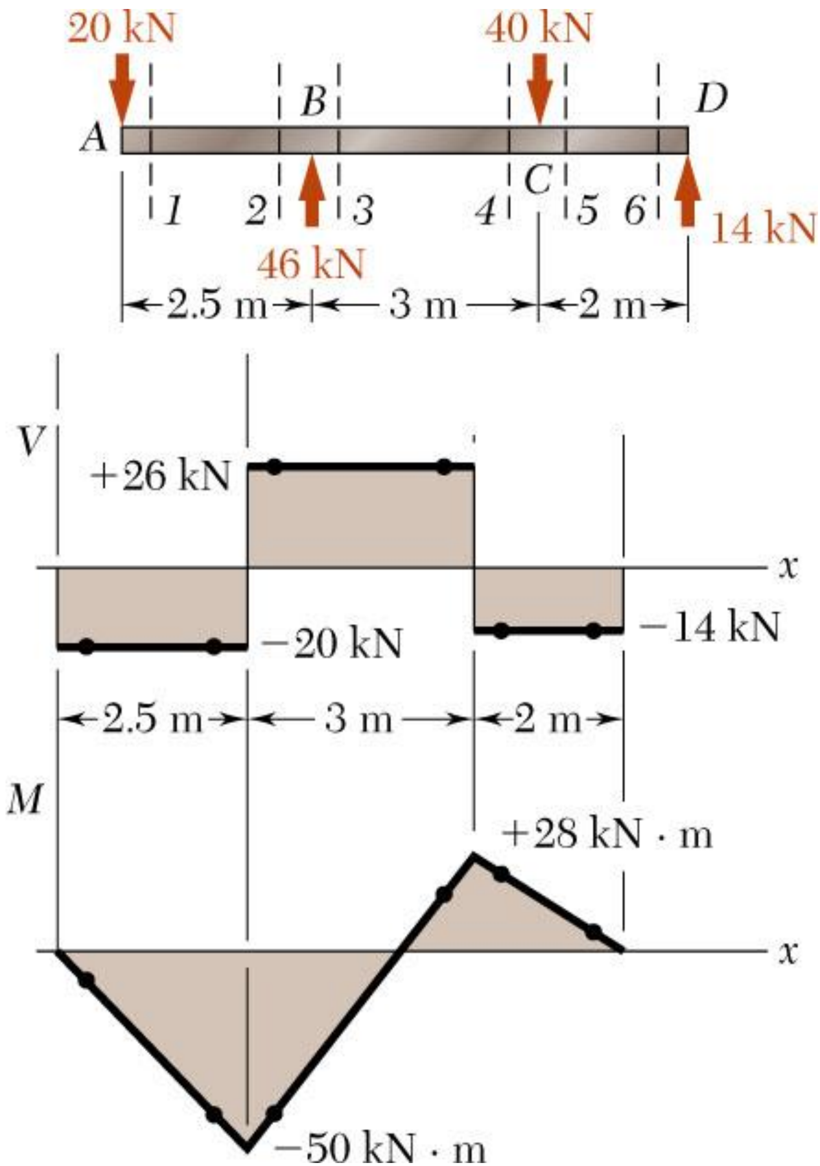
SOLUTION:

- Treating the entire beam as a rigid body, determine the reaction forces
- Section the beam at points near supports and load application points. Apply equilibrium analyses on resulting free-bodies to determine internal shear forces and bending couples
- Identify the maximum shear and bending-moment from plots of their distributions.
- Apply the elastic flexure formulas to determine the corresponding maximum normal stress.

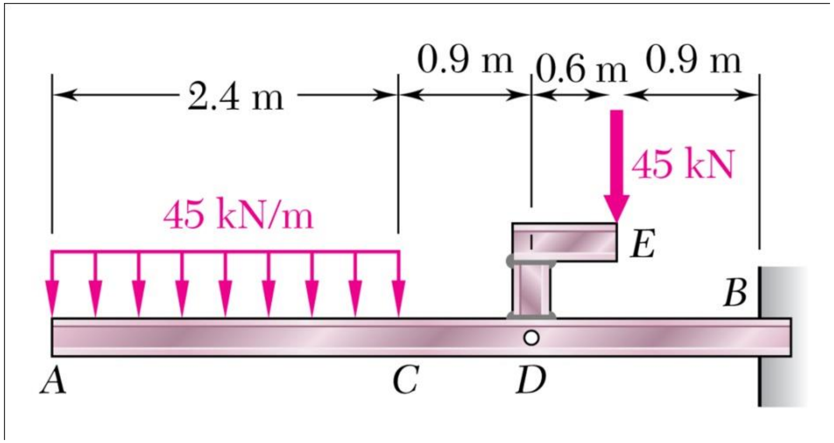
Sample Problem 5.1



Sample Problem 5.1



Sample Problem 5.2

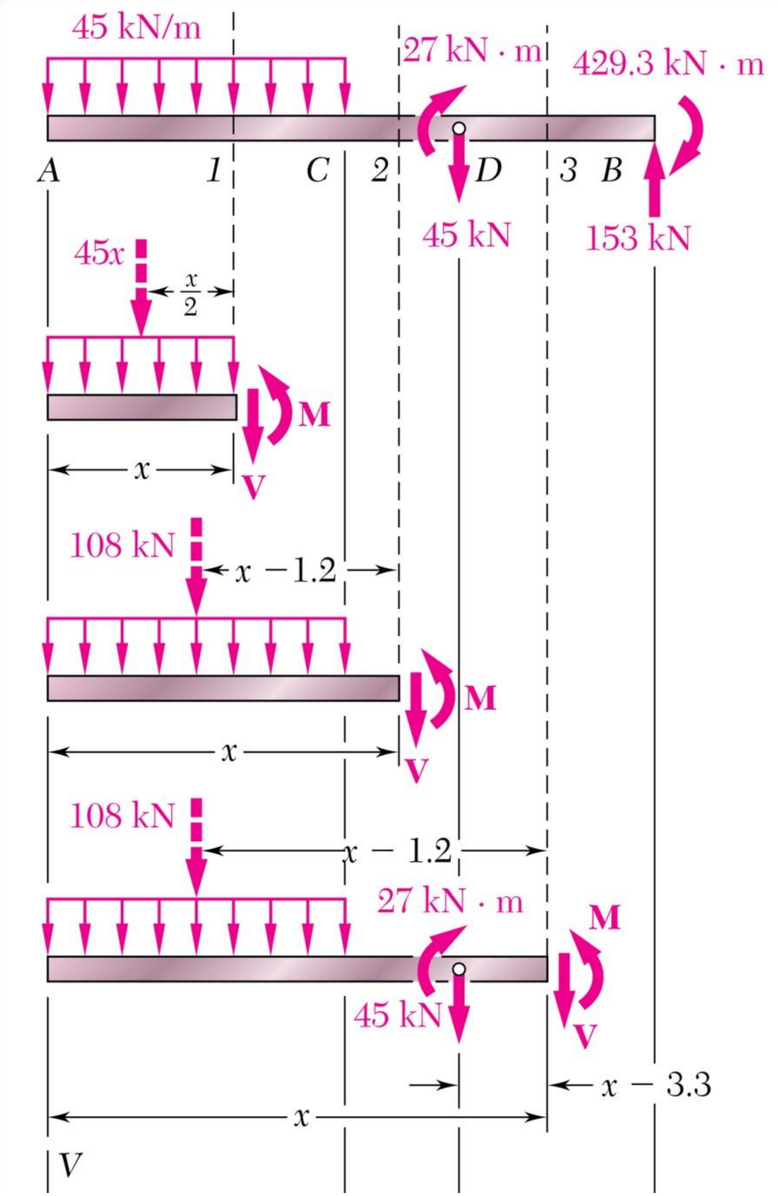


The structure shown is constructed of a W 250x167 rolled-steel beam. (*a*) Draw the shear and bending-moment diagrams for the beam and the given loading. (*b*) determine normal stress in sections just to the right and left of point *D*.

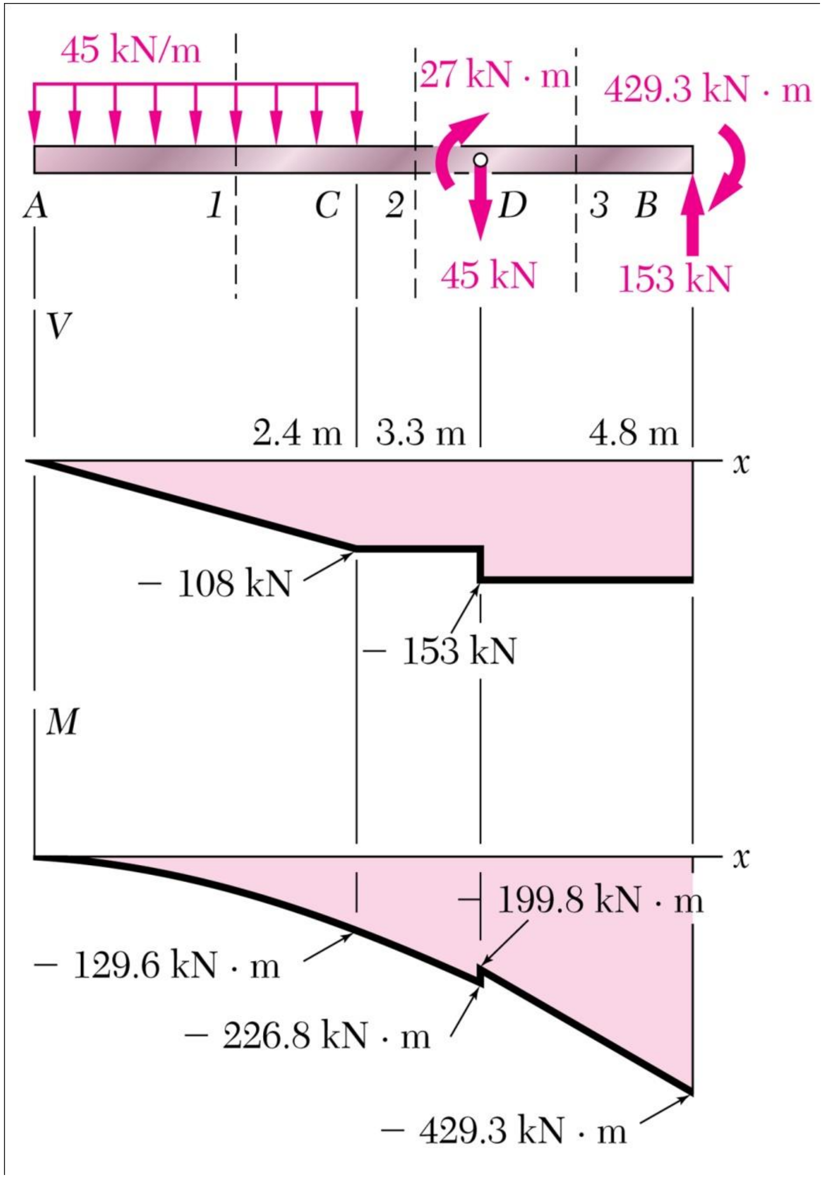
SOLUTION:

- Replace the 45 kN load with an equivalent force-couple system at *D*. Find the reactions at *B* by considering the beam as a rigid body.
- Section the beam at points near the support and load application points. Apply equilibrium analyses on resulting free-bodies to determine internal shear forces and bending couples.
- Apply the elastic flexure formulas to determine the maximum normal stress to the left and right of point *D*.

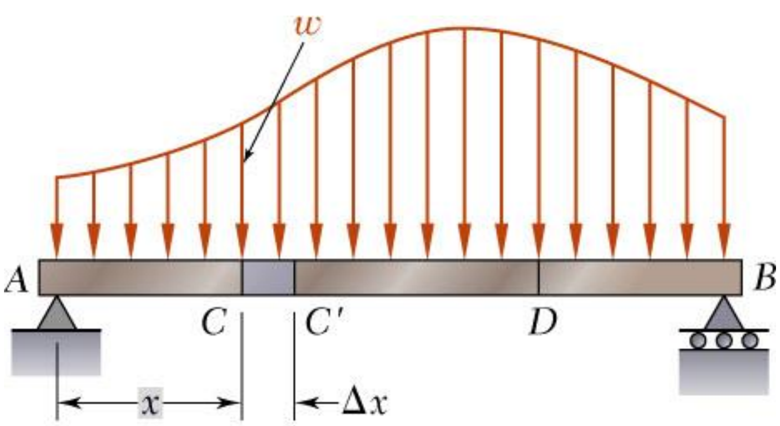
Sample Problem 5.2

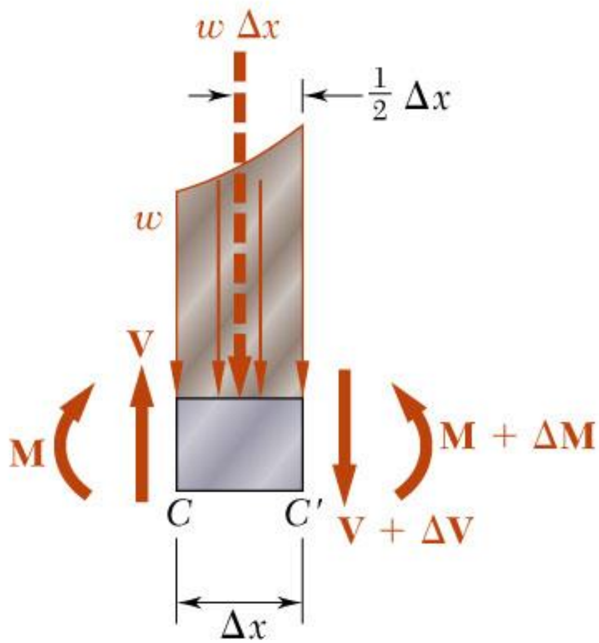


Sample Problem 5.2



5.2 Relations Among Load, Shear, and Bending Moment





- Relationship between load and shear:

$$\sum F_y = 0: \quad V - (V + \Delta V) - w\Delta x = 0$$

$$\Delta V = -w\Delta x$$

$$\frac{dV}{dx} = -w$$

$$V_D - V_C = -\int_{x_C}^{x_D} w\, dx$$

- Relationship between shear and bending moment:

$$\sum M_{C'} = 0: \quad (M + \Delta M) - M - V\,\Delta x + w\Delta x \frac{\Delta x}{2} = 0$$

$$\Delta M = V\,\Delta x - \tfrac{1}{2} w (\Delta x)^2$$

$$\frac{dM}{dx} = V$$

$$M_D - M_C = \int_{x_C}^{x_D} V\, dx$$

- P355 5.7, 5.9

Concept Application 5.3

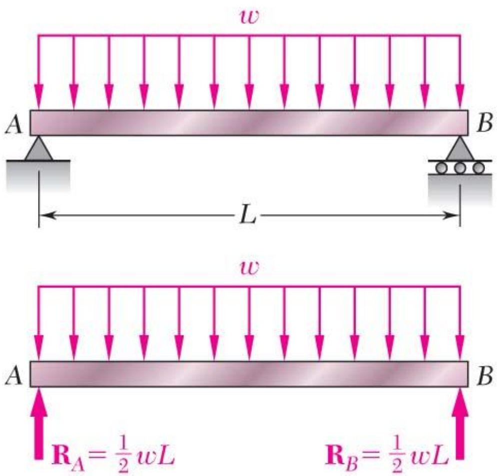


Fig. 5.13

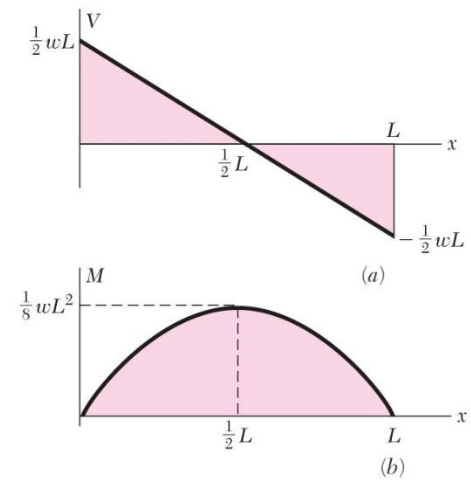
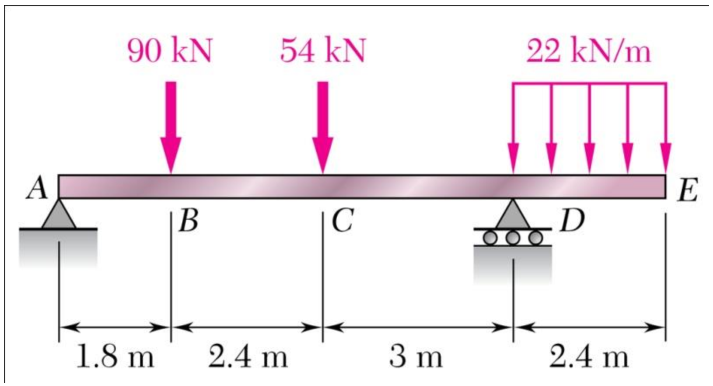


Fig. 5.14

Sample Problem 5.3

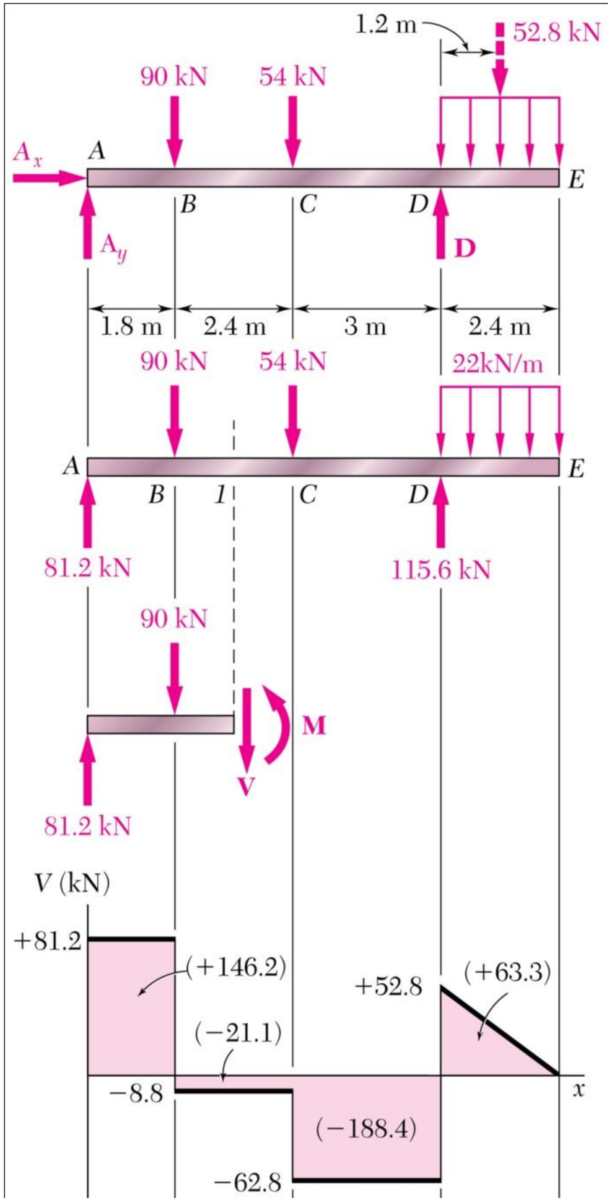


Draw the shear and bending moment diagrams for the beam and loading shown.

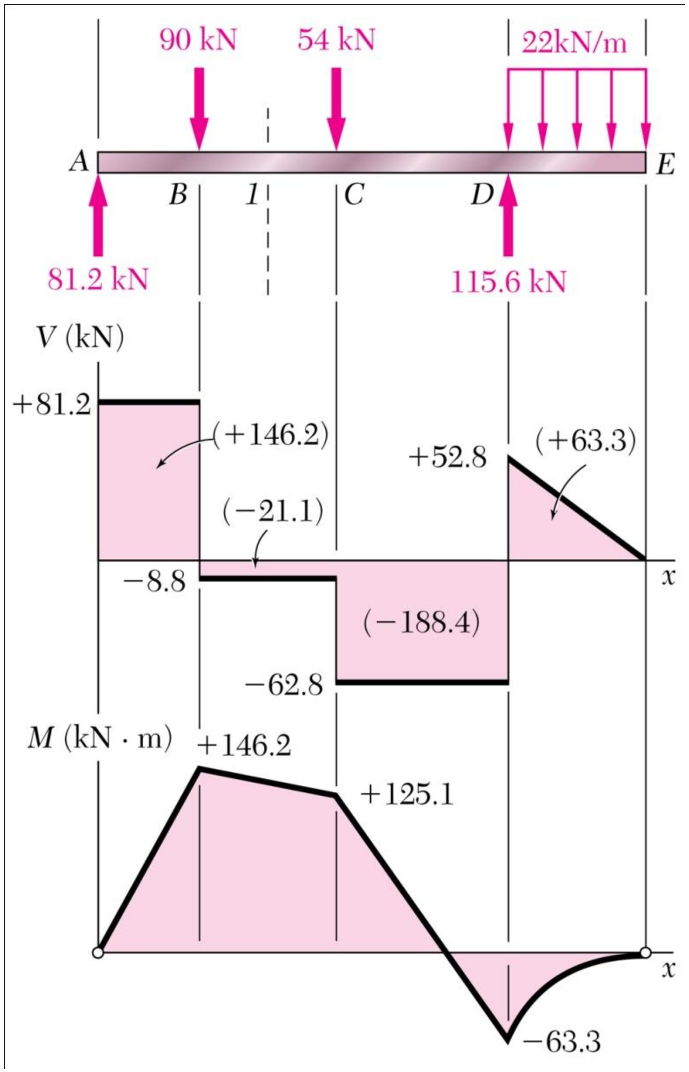
SOLUTION:

- Taking the entire beam as a free body, determine the reactions at *A* and *D*.
- Apply the relationship between shear and load to develop the shear diagram.
- Apply the relationship between bending moment and shear to develop the bending moment diagram.

Sample Problem 5.3



Sample Problem 5.3

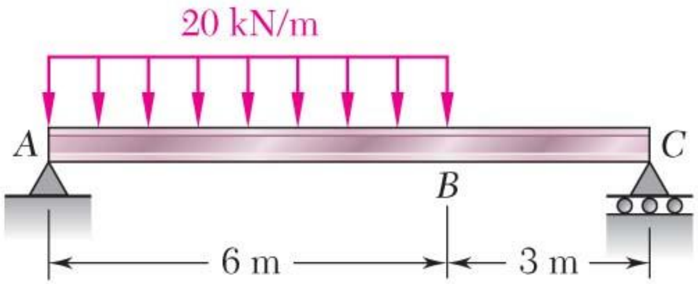


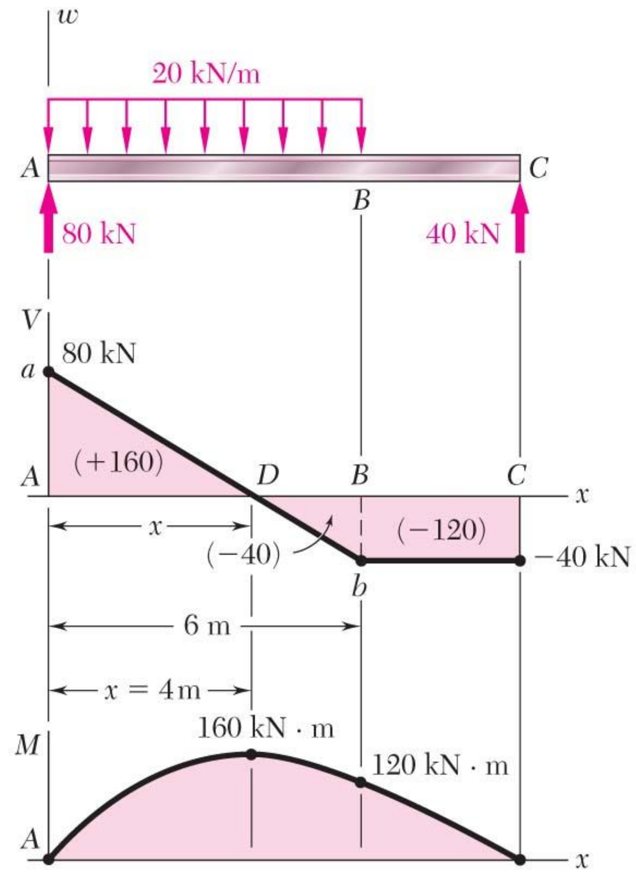
- Apply the relationship between bending moment and shear to develop the bending moment diagram.

$$\frac{dM}{dx} = V \qquad dM = V\,dx$$

- bending moment at *A* and *E* is zero
- bending moment variation between *A*, *B*, *C* and *D* is linear
- bending moment variation between D and *E* is quadratic
- net change in bending moment is equal to areas under shear distribution segments
- total of all bending moment changes across the beam should be zero

Sample Problem 5.4





- P368 5.51

5.3 Design of Prismatic Beams for Bending

- The largest normal stress is found at the surface where the maximum bending moment occurs.

$$\sigma_m = \frac{|M|_{\max} c}{I} = \frac{|M|_{\max}}{S}$$

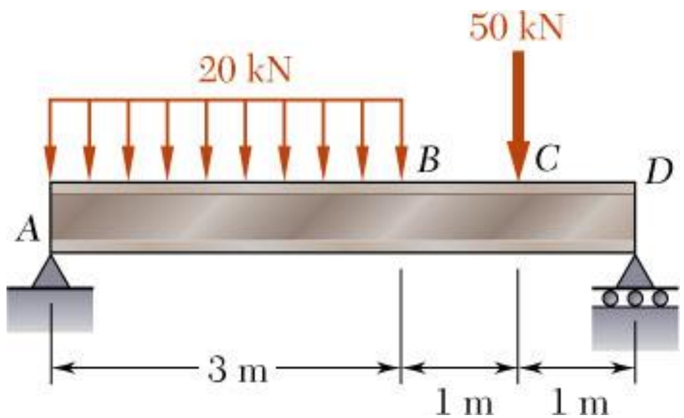
- A safe design requires that the maximum normal stress be less than the allowable stress for the material used. This criteria leads to the determination of the minimum acceptable section modulus.

$$\sigma_m \le \sigma_{all}$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{all}}$$

- Among beam section choices which have an acceptable section modulus, the one with the smallest weight per unit length or cross sectional area will be the least expensive and the best choice.

Sample Problem 5.8

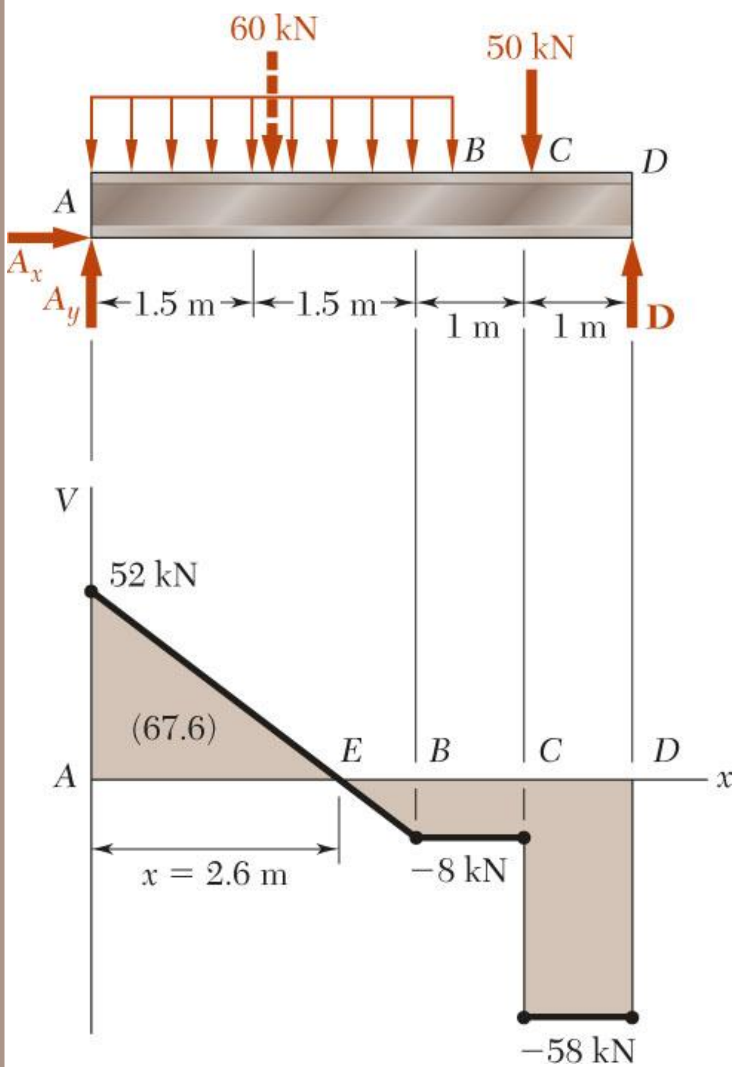


A simply supported steel beam is to carry the distributed and concentrated loads shown. Knowing that the allowable normal stress for the grade of steel to be used is 160 MPa, select the wide-flange shape that should be used.

SOLUTION:

- Considering the entire beam as a free-body, determine the reactions at *A* and *D*.
- Develop the shear diagram for the beam and load distribution. From the diagram, determine the maximum bending moment.
- Determine the minimum acceptable beam section modulus. Choose the best standard section which meets this criteria.

Sample Problem 5.8



Sample Problem 5.8

Shape	$S \times 10^3 \text{mm}^3$
W410×38.8	637
W360×32.9	474
W310×38.7	549
W250×44.8	535
W200×46.1	448

- Choose the best standard section which meets this criteria.

$\boxed{W360 \times 32.9}$