# 05 Advance Analysis 多步驟分析 / 平面應變&平面應力 非線性分析(接觸&材料設定) / 結構最佳化

### **2D Plane Problem**

Plane stress, plane strain, axisymmetric, plane stress w/thk

Plane stress



### **2D Plane Problem**

#### Plane strain

#### The cross-sections are along the thickness





### **2D Plane Problem**

#### Axisymmetric

#### The cross-sections are along the radial directions





### Plane Stress – Ex.10

The bicycle wrench shown in figure is made of steel with 3 mm thick. Determine the von-Mises stresses under the given distributed load and boundary conditions.



### Plane Stress – Ex.10



The bicycle wrench shown in figure is made of steel with 3 mm thick. Determine the von-Mises stresses under the given distributed load and boundary conditions.









#### Stresses in a Long Cylinder

A long thick-walled cylinder (with inner radius = 4 in. and outer radius = 8 in.) made of steel is initially subjected to an constant pressure (of 30000 psi). The pressure is then removed and the cylinder is subjected to a constant rotation (60000 rpm) about its center line. Find the radial displacement, radial stress, and hoop stress at the two load steps. (Face mesh size = 0.25 in)



### Plane Strain – Ex.11-1

#### Stresses in a Long Cylinder

A long thick-walled cylinder (with inner radius = 4 in. and outer radius = 8 in.) made of steel is initially subjected to an constant pressure (of 30000 psi). The pressure is then removed and the cylinder is subjected to a constant rotation (60000 rpm) about its center line. Find the radial displacement, radial stress, and hoop stress at the two load steps. (Face mesh size = 0.25 in)

#### <u>Step. 1</u>





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#### Load Steps

- In a linear static or steady-state analysis, you can use different load steps to apply different sets of loads--wind load in the first load step, gravity load in the second load step, both loads and a different support condition in the third load step, and so on.
- In a transient analysis, multiple load steps apply different segments of the load history curve.



### Introduction of ANSYS – Solution

#### Substeps

- Substeps are points within a load step at which solutions are calculated. You use them for different reasons:
  - In a linear or nonlinear transient analysis, use substeps to satisfy transient time integration rules
  - In a nonlinear static or steady-state analysis, use substeps to apply the loads gradually so that an accurate solution can be obtained.



### Introduction of ANSYS – Solution



### Step Controls

# Project i Model (A4) i √ Geometry i √ Materials · √ Materials · √ Mesh



Details of "Analysis Settings" 🔷 🔻 🗖 🗖 🗙					
	Step Controls				
	Number Of Steps	2.			
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_	Auto Time Stepping	Program Controlled	Ξ		
-	Solver Controls				
	Solver Type	Program Controlled			
	Weak Springs	Off			
	Solver Pivot Checking	Program Controlled			
	Large Deflection	Off			
	Inertia Relief	Off			
+	Rotordynamics Controls				
+	Restart Controls				



### Plane Strain – Ex.11-2

#### Stresses in a Long Cylinder

A long thick-walled cylinder (with inner radius = 4 in. and outer radius = 8 in.) made of steel is initially subjected to an constant pressure (of 30000 psi). The pressure is then removed and the cylinder is subjected to a constant rotation (60000 rpm) about its center line. Find the radial displacement, radial stress, and hoop stress at the two load steps. (Face mesh size = 0.25 in)

#### <u>Step. 2</u>





學習目標

圓柱座標轉換 Face Meshing 角速度設定 多步驟分析

### **Nonlinear Structural Analysis**

- 有限元素法中,常見的非線性(Nonlinear)問題主要可分為三類
  - ➢ 幾何非線性(Geometric nonlinearity)
    - ✓ 大變形、大位移





- ▶ 材料非線性(Material nonlinearity)
  - ✓ 非線性行為之材料特性,即應力應變曲線非線性關係



- ▶ 邊界非線性(Contact analysis)
  - ✓ 非線性之接觸行為,如: Frictional、Frictionless、Rough

設定<mark>Engineering Data</mark>,指定 為非線性材料性質



- 若應力分析範圍大於降伏強度,因應力與應變呈曲線關係,便成為材料非線 性分析,這和遵循虎克定律之線彈性分析完全不同
- 在結構分析中,若負荷過程的應力包含了彈性部分與超過強伏點的塑性部分, 可稱為彈塑性(elastoplastic)分析



材料 亍為	變形 型式	與應變率 (或時間) 之關 係 (Rate dependency)	材料性質分類	材料模型 (Material laws)
<u></u> 泉性	彈性	無關 (Rate-independent)	Linear elastic	Hooke's law
				Mooney-Rivlin
		毎月 (Pata independent)	Hyperelastic	Arruda-Boyce
	彈性	High (Nate-Independent)		Blatz-Ko
- - 線性			Multilinear elastic	Multilinear elastic
		相關 (Rate-dependent)	Viscoelasticity	Viscoelasticity
				Bilinear isotropic
			leatronic hardoning placticity	Multilinear isotropic
	非彈性	無關 (Rate-independent)	isou opic nardening plasucity	Voce's nonlinear isotropic
				Anisotropic
				Bilinear kinematic
線性			Kinematic hardening plasticity	Multilinear kinematic
				Chaboche
			Combined kinematic and instrumin	Chaboche and bilinear isotropic
			bardening plasticity	Chaboche and multilinear isotropic
			naruening plasticity	Chaboche and Voce's
			Pressure-dependent plasticity	Druger-Prager
			Visconlacticity	Creep
			viscopiasticity	Anand
		相關 (Rate-dependent)	Combined areas and jestropia	Creep and bilinear isotropic
			bardening plasticity	Creep and multilinear isotropic
			naraening plasuoity	Creep and Voce's 14

### Material Nonlinearity – 彈塑性分析

- 塑性力學(plasticity)所研究的對象是延性材料在降伏後(應力>S<sub>y</sub>)所發生的 塑性變形,屬於材料非線性分析的一種,由圖可看出結構受力過程包括了彈 性與塑性變形,這類分析稱為彈塑性(elastoplastic)分析
- ■材料達到塑性後·主要現象為產生應變硬化(strain hardening)與永久變形
- 若材料達到塑性變形(應力>S<sub>y</sub>),當外力移除後將留下永久變形,圖中之<sub>p</sub> 即為移除外力後所殘留之塑性應變(plastic strain),且在許多情況下會有殘 留應力(residual stress)存在於材料內部



### Material Nonlinearity – 彈塑性分析

- 相對的在數學解析或有限元素分析上,有以下幾種應力應變曲線模式:
  - 1. 彈性-完全塑性(elastic perfectly-plastic)
  - 2. 雙線性(bi-linear)
  - 3. 多線段(multi-linear)
  - 4. 塑性曲線



Α Static Structural 21 2 🥏 Engineering Data 3 Geometry < 🖌 ✓ 🖌 Model 4 ? 🍓 Setup 5 7\_ Solution 6 7 🥪 Results 7

#### ■ Engineering Data(工程資料)

٨			Unsaved Project - Workbench	Ì			- 0 ×
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🗆 Linear Elastic	1	Contents of Engineering Data 🗦 🐼 S.,	. Description	1	Temperature (C) 🚊 Poisson's Ratio	-	
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Experimental Stress Strain Data			2, Table 5-110.1				
🔁 Uniaxial Test Data	4	Sector Contical bone					
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	4	Events Understanding Hardware Selection for Struc	tural Mechanics				
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### Nonlinear – Ex.12-1

#### 殘留應力分析

如圖所示類似彈簧片的結構,其尺寸為I=0.02m,h=0.015m,a=0.02m,b=0.01m,圓角 R=0.001m,厚度t=0.001m。邊界條件如圖,長度a之底面部分全部拘束固定,當給定之y方向位 移dy=-0.002m時,該結構若有塑性變形,求出移除負荷後的殘留變形。

(1)材料為**bilinear material**、楊氏模數E=210000×10<sup>6</sup>Pa、其真應力應變曲線如圖之BISO曲線、 $E_T=30000\times10^6$ Pa、普松比v=0.3、降伏強度S<sub>v</sub>=200×10<sup>6</sup>Pa。  $\sigma$ 







### Nonlinear – Ex.12-1

bilinear material



Type: Equivalent Total Strain Unit: m/m Time: 2 2023/2/14 下午 03:40

0.002217 Max 0.0019707 0.0017244 0.001478 0.0012317 0.00098535 0.00073901 0.00049268 0.00024634 2.2132e-13 Min

Step. 2



#### 殘留總應變(彈性+塑性)

等效總應變 Equivalent Total Strain



等效應力 Equivalent Stress



#### 殘留應力分析

如圖所示類似彈簧片的結構,其尺寸為I=0.02m,h=0.015m,a=0.02m,b=0.01m,圓角 R=0.001m,厚度t=0.001m。邊界條件如圖,長度a之底面部分全部拘束固定,當給定之y方向位 移d<sub>y</sub>=-0.004m時,該結構若有塑性變形,求出移除負荷後的殘留變形。

(2)材料為multilinear material,數據採用自文獻,楊氏模數E=71200MPa,普松比v=0.31,降 伏強度S<sub>y</sub>=260MPa,真應力應變曲線原為一實際曲線,經MISO之多直線段簡化且輸入ANSYS後 如圖,各點座標如下所示。



multilinear material

1 (0, 260) 2 (0.0066, 341.68) 3 (0.0083, 358.34) 4 (0.0133, 375.01) 5 (0.0266, 408.34) 6 (0.0566, 441.68)





### Nonlinear – Ex.12-2

#### Step. 2 multilinear material

Type: Equivalent Total Strain Unit: m/m Time: 2 2023/4/25 下午 11:09

0.0001826 Max 0.00016231 0.00014202 0.00012173 0.00010145 8.1156e-5 6.0867e-5 4.0578e-5 2.0289e-5 7.6132e-17 Min

Type: Equivalent (von-Mises) Stress Unit: Pa Time: 2 2023/4/25 下午 11:08 4.585e6 Max 4.0756e6 3.5661e6 3.0567e6 2.5472e6 2.0378e6 1.5283e6 1.0189e6 5.0945e5 5.4144e-6 Min



#### 殘留總應變(彈性+塑性)

等效總應變 Equivalent Total Strain



等效應力 Equivalent Stress

### **Contact Analysis**







- 有限元素接觸分析是於物體之間的接觸面上加入非貫穿(non-penetration) 條件
- 以下圖之兩物體接觸為例,首先須將Sa和Sb兩個面定義為接觸面,下令兩 接觸面不可貫穿,如此一來,只要圓形物體受力變形,便可透過兩接觸面Sa 和Sb將力量傳至矩形物體,使得矩形物體也跟著變形,即完成了接觸分析
- ANSYS是利用接觸元素(contact elements)來模擬接觸面,只要接觸區域 的接觸元素一被建立,計算時就會考慮到接觸條件



### **Contact Analysis**

ELEMENTS



### **Contact Analysis**



■ 在ANSYS定義中,目標面(Target surface)的節點可以穿透接觸面(Contact surface),接觸面的節點則不可穿透目標面

#### ■ 接觸對之接觸元素和目標元素建立原則



目標面(Target surface)	接觸面(Contact surface)
網格較粗	網格較細
剛性較大(硬)	剛性較小(軟)
面積顯著較大(平面、凹面)	面積顯著較小(尖、凸面)





資料來源https://www.youtube.com/watch?v=kQgYAisb5V4&list=PLF1WR5I3KTKQ4IQkA56C1m-FU40mB1mzz&index=3

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Static Structural

Finding Data

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- 介面條件設定
  - 模型為組合件時,若要組合件間能共用面,達到力量直接傳遞時,必須將此部份 組件形成一個群組>From New part
  - ➢ 若要組合件間能有各自的面,達到contact效果時,則不須進行此動作,模型匯入 後軟體會自動判斷出非連續面之部份



- 接觸(Contact)設定
  - ▶ 軟體會自動偵測到不同part之界面 · 並於Connection中顯示所有之contact區域
  - ▶ 接觸行為設定
    - ✓ Bonded
      - 預設項目,沒有相對滑動和分離,會忽略初始穿刺(penetration),模擬為相互連接
    - ✓ No Separation
      - •類似Bonded,僅適用於面(3D)或邊(2D)之接觸,沒有相對分離,僅可延接觸面有些微無摩擦滑動
    - ✓ Frictionless
      - 此為單邊接觸,假設摩擦係數為0,允許相對滑動,出現分離時法向量壓力為0,法向會分離
    - ✓ Rough
      - 類似Frictionless,有摩擦係數,無相對滑動,法向會分離
    - Frictional
      - 有摩擦係數,有相對滑動,法向會分離





A Static Structural

Engineering Data

Geometry

Model

🍓 Setup

Solution

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### Contact – Ex.13



#### 人工牙根分析

請構出硬質骨與鬆質骨圓柱模型,尺寸如圖所示,並將外部CAD軟體建構出之植體檔(screw.igs) 匯入,施加側向力200N負載於植體頂部(已於植體頂部建構一凹點特徵)上,並設定硬質骨/鬆質骨 外側自由度為0(如下圖)。各材料特性:硬質骨(楊氏係數17000MPa;蒲松比0.3)、鬆質骨(楊氏係 數200MPa;蒲松比0.2)及植體(鈦合金楊氏係數110000MPa;蒲松比0.33),採用四面體網格,網 格尺寸:植體0.5mm、硬質骨0.8mm、鬆質骨1.0mm、硬質骨內側螺紋面0.5mm。觀察硬質骨最 大主應變(Maximum Principal strain)及植體最大等效應力(von-Mises stress)情形。

(1) 植體與硬質骨/鬆質骨界面未結合(unbonded)狀態之設定(模擬植體剛植入骨頭)

(2) 植體與硬質骨/鬆質骨界面結合(bonded)狀態之設定(模擬植體與骨頭已骨整合)



### Contact – Ex.13

#### (1) Unbonded

Type: Equivalent (von-Mises) Stress Unit: MPa Time: 1 2023/4/25 下午 09:55 5450.3 Max

4844.7 4239.1 3633.5 3028 2422.4 1816.8 1211.3 605.71 0.14336 Min

 Type: Maximum Principal Elastic Strain

 Unit: mm/mm

 Time: 1

 2023/4/25 下午 10:01

 0.0063057 Max

 0.0056054

 0.0042047

 0.0035043

 0.002804

 0.0021037

 0.0014033

 0.00070297

 2.6203e-6 Min

Max



#### Screw

等效應力 Equivalent Stress

#### **Cortical Bone**

最大主應變 Maximum Principal Strain

### Contact – Ex.13

#### (2) Bonded







#### Screw

等效應力 Equivalent Stress

#### **Cortical Bone**

最大主應變 Maximum Principal Strain



#### 非線性材料

如圖為一冰箱門封元件,由兩鋼板及一長條形封條組成。封條為超彈性材料,其材料特性由實驗 量測得到(TESTDATA),包含單軸/雙軸拉伸測試及剪力測試。

本習題將學習如何藉由實驗數據輸入得到超彈性材料特性,並模擬封條受兩鋼板擠夾之力學行為。此次將以2D進行模型建構,並以PLANE STRAIN進行模擬後觀察其最大主應力(變)/最小主應力(變)/剪應力(變)。

#### 位移0.85"



### Nonlinear & Contact – Ex.14

學習目標 •材料data匯入 •Hyperelastic



2D模型如圖所示,其為剛性接觸的兩物體且下端整面為固定並於上端

邊線受一5MPa之壓力,請針對該模型進行接觸剛性分析。







■ 拓樸最佳化(Topology Optimization)是一種結構優化技術,可自行定義設計範圍(Design Domain),根據給定的限制條件與目標函數,解出符合給定條件之最佳結構



- 五楼最佳化結合有限元素法及最佳化演算法,將最佳密度值分配給定義域中的每個元素,求得該結構之最佳材料分布情形
- 可實現在一定的結構強度要求下,將材料做出最有效之配置,以節省不必要 的材料達到輕量化之目標



Sutradhar et. al., 2017

圖片來源: https://courses.ansys.com/index.php/courses/topology-optimization-using-ansys-mechanical/lessons/topology-optimization-of-a-bell-crank-lesson-1/

### Topology Optimization (Structural Optimization)

**Optimization Region** 

Iteration Number: N/A

2023/2/25 上午 07:45

Exclusion Region

- ➢ 設計範圍 Optimization Region
  - ✓ Design Region
  - ✓ Exclusive Region



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	Design Region				
	Scoping Method	Geometry Selection			
	Geometry	All Bodies			
	Exclusion Region				
	Define By	Boundary Condition			
	Boundary Condition	All Boundary Conditions			
	Definition				
	Suppressed	No			
Optimization Option     Optimization Type     Topology Optimization - Density Based					





### Topology Optimization (Structural Optimization)

- ▶ 目標函數 Objective
- ▶ 限制條件 Response Constraint

#### Objective



#### В Structural Optimization 1 \_ 2 Engineering Data ? 🖌 3 Geometry ?, Model ? 5 Setup ? 🖌 6 6 Solution ?. 9 7 Results

### Structural Optimization (Topology Optimization)

Remove (0.0 to 0.4)

Keep (0.6 to 1.0)

▶ 最佳結構 - Topology Density(拓樸密度)



•		В			
1		Structural Optimization			
2	٢	Engineering Data	$\checkmark$	4	
3	$\bigcirc$	Geometry	?	4	
4	۲	Model	7	4	
5	٢	Setup	7	4	
6	(i)	Solution	7	4	
7	9	Results	7	4	
				_	

Details of "Topology Density"						
-	Scope		^			
	Scoping Method	Optimization Region				
	Optimization Region	Optimization Region				
-	Definition					
	Туре	Topology Density				
	Ву	Iteration				
	Iteration	Last				
	Retained Threshold	0.5				
	Exclusions Participation	Yes				
	Calculate Time History	Yes				
	Suppressed	No				
-	Results					
	Minimum	1.e-003				
	Maximum	1.	v			



### **Structural Optimization – Ex.16**



平板模型如圖所示,請針對該模型進行拓撲最佳化(Topology Optimization),最佳化目標為剛度 最大情形下,減少40%體積。材料使用Structural steel,網格形式採用四面體(Tetrahedrons), 網格尺寸為1mm,平板左側固定並於圓孔曲面上施以力量(Y方向,向下10N)。





Unit : mm

### **Structural Optimization – Ex.16**



Equivalent Stress Type: Equivalent (von-Mises) Stress Unit: MPa Time: 1 s 2023/2/25 上午 07:02





#### 等效應力 Equivalent Stress

Topology Density Type: Topology Density Iteration Number: 20 2023/2/25 上午 07:05





拓樸密度結構 Topology Density

### **Design Validation – Ex.17**



三角bracket模型如圖所示,請針對該模型右側區塊進行拓撲最佳化(Topology Optimization), 最佳化目標為剛度最大情形下,減少50%體積。材料使用Structural steel,網格尺寸為1mm, 左側固定,右側圓孔曲面上施以力量,並利用Design Validation將最佳化結構再次進行力學分析 。試比較不同力量下,最佳化結果與結構應力分析結果差異。

- 1) Y方向,向下100N
- 2) X方向·向右100N



100N

100N

### **Design Validation – Ex.17**





Type: Equivalent (von-Mises) Stress Unit: MPa Time: 1 s 2023/3/23 上午 07:59





#### 原始模型等效應力(向下) Equivalent Stress

原始模型等效應力(向右) Equivalent Stress



Remove (0.0 to 0.4) Marginal (0.4 to 0.6) Keep (0.6 to 1.0)





Type: Topology Density Iteration Number: 10 2023/3/23 上午 07:59

Remove (0.0 to 0.4) Marginal (0.4 to 0.6) Keep (0.6 to 1.0)



### **Design Validation – Ex.17**





Type: Equivalent (von-Mises) Stress Unit: MPa Time: 1 s 2023/3/23 上午 07:59





#### 原始模型等效應力(向下) Equivalent Stress









