材料力學

Mechanics of Materials

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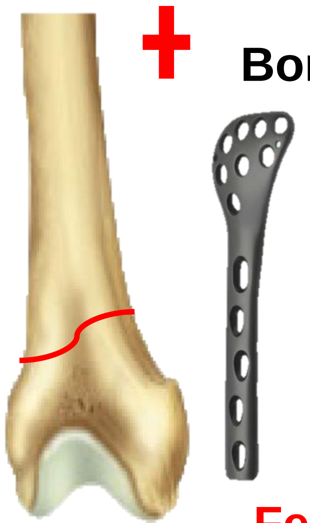
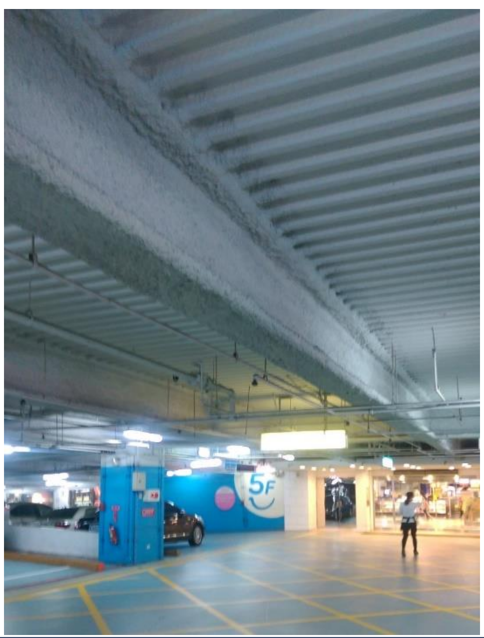


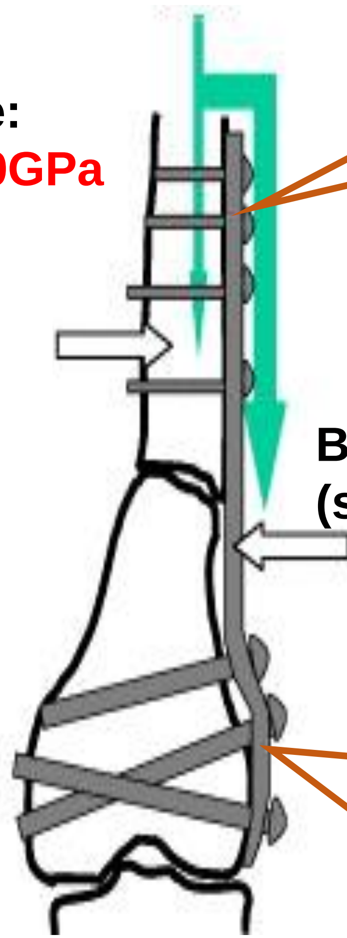
Bone plate

Cortical bone:
10-20GPa

Femur fracture



Load



Bone plate
(stainless st
200GPa

Cross section

b

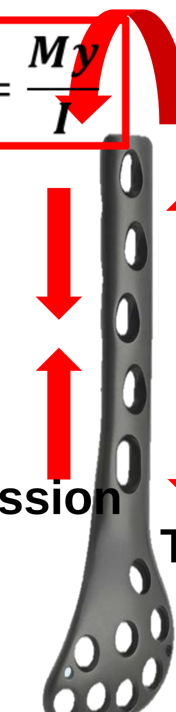
h

Moment inertia

$$I = \frac{b * h^{\wedge}3}{12}$$

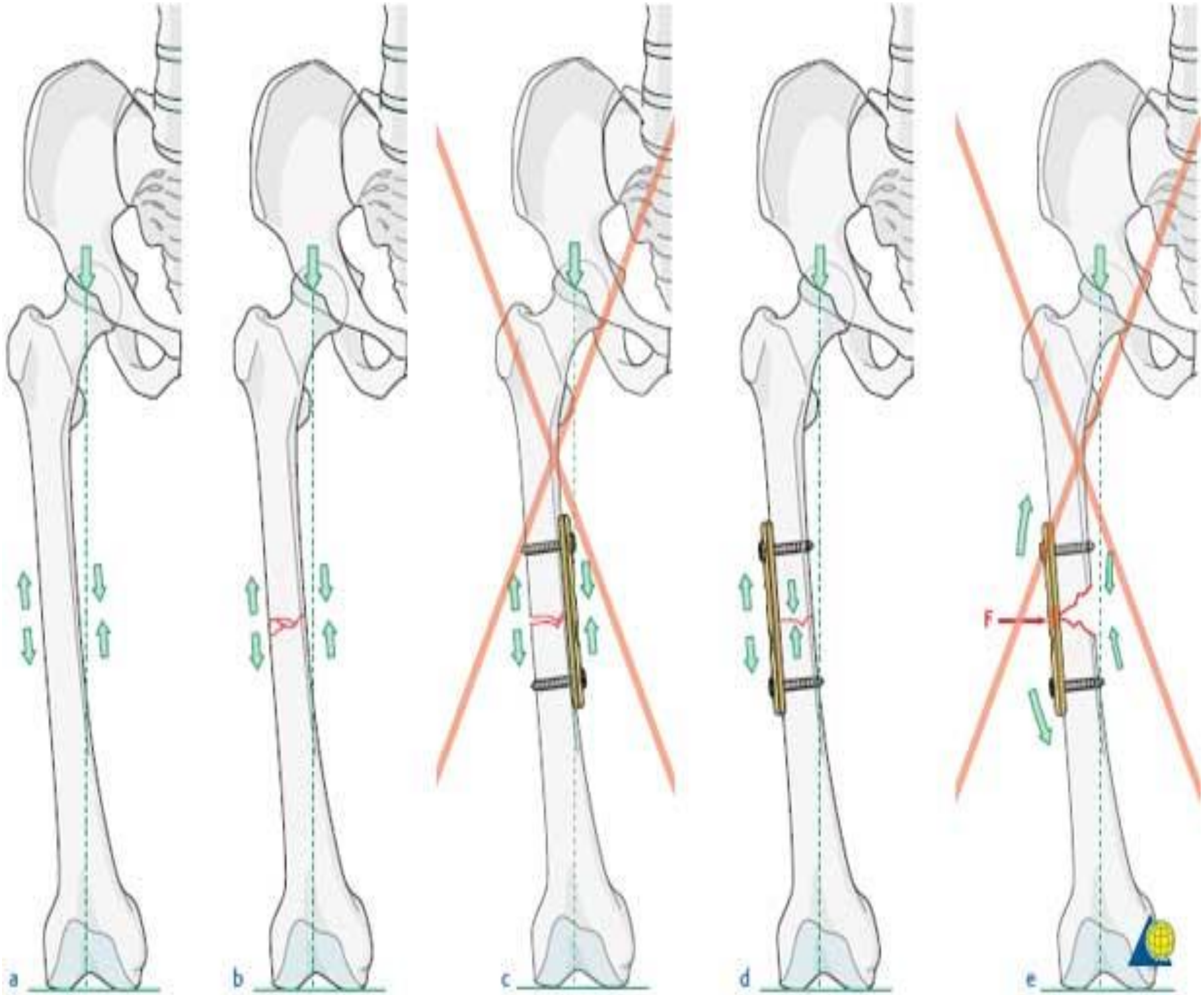
Bending

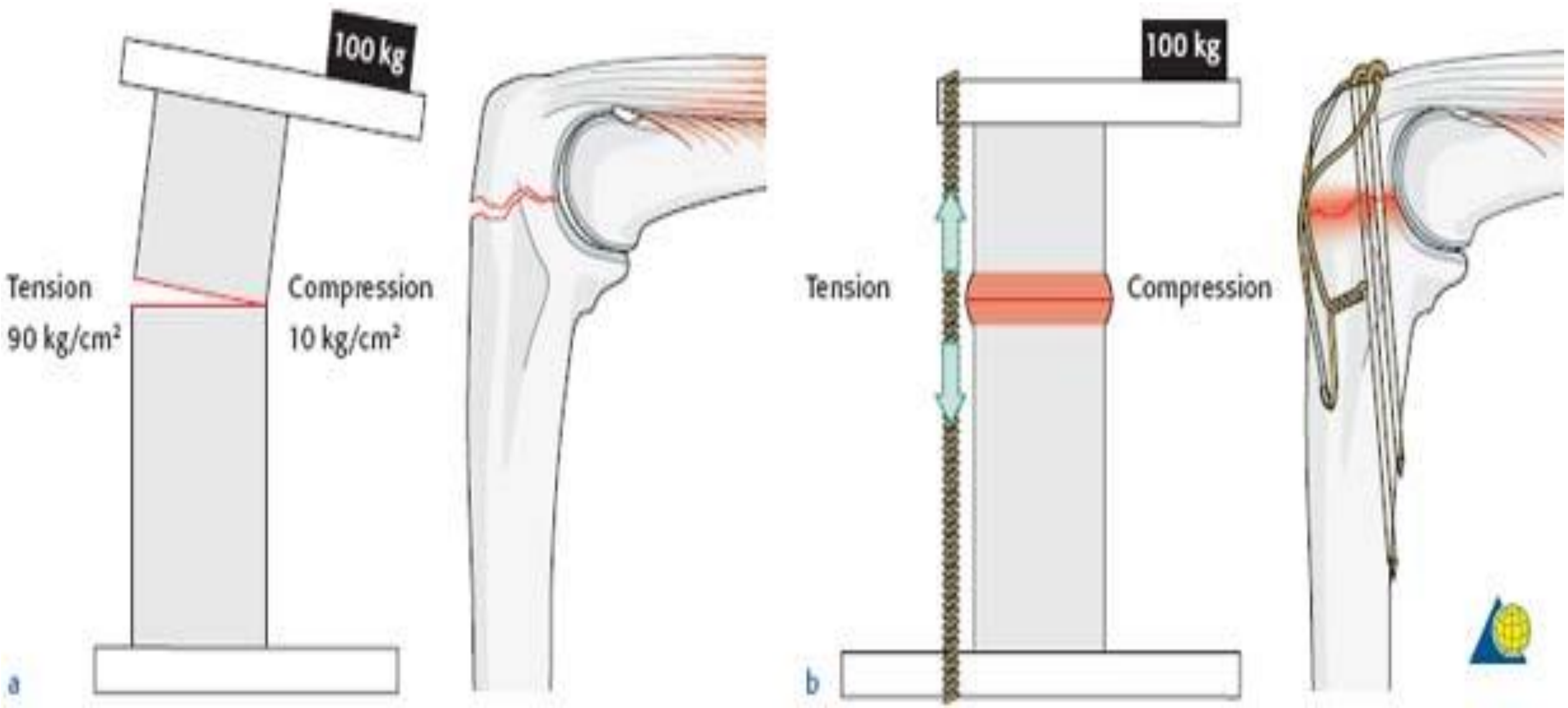
$$stress = \frac{My}{I}$$



Compression

Tension


a
b
c
d
F
e

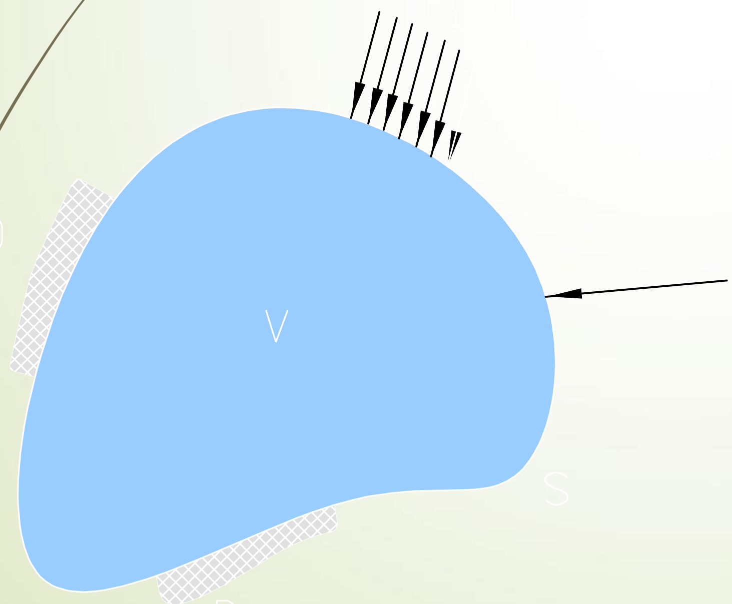

100 kg
Tension
90 kg/cm²
Compression
10 kg/cm²
a
100 kg
Tension
Compression
b

Basic Mechanics

- Force
  - Force can be defined as mechanical disturbance or load. When an object is pushed or pulled, a force is applied on it.
  - A force acting on an object may deform the object, change its state of motion, or both.
  - A force may be internal or external, normal (perpendicular) or tangential; tensile, compressive, or shear; gravitational (weight); or frictional.

Basic Mechanics

- Description of Structural Analysis Problems
  - 結構分析是針對一個物體（body）在一定的負載（loads）下，去探索其力學上的反應（responses）



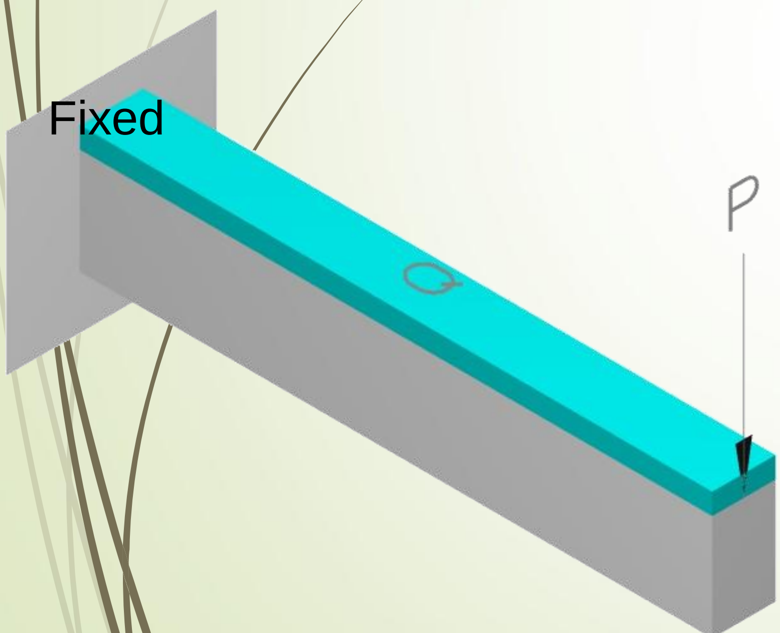
- Distributed forces
- Concentrated force
- Fixed or Displacements
- Free surface

Basic Mechanics

- Description of Structural Analysis Problems

P:concentrated force

Q:distributed forces



<table>
<tr><th colspan="3">負載分類</th></tr>
<tr><td rowspan="4">作用在物體邊界</td><td rowspan="2">力</td><td>分佈力</td></tr>
<tr><td>集中力</td></tr>
<tr><td rowspan="2">變位</td><td>零變位</td></tr>
<tr><td>非零變位</td></tr>
<tr><td rowspan="3">作用在物體內部</td><td rowspan="2">力</td><td>慣性力
(重力或離心力)</td></tr>
<tr><td>其他分佈力
(電磁力)</td></tr>
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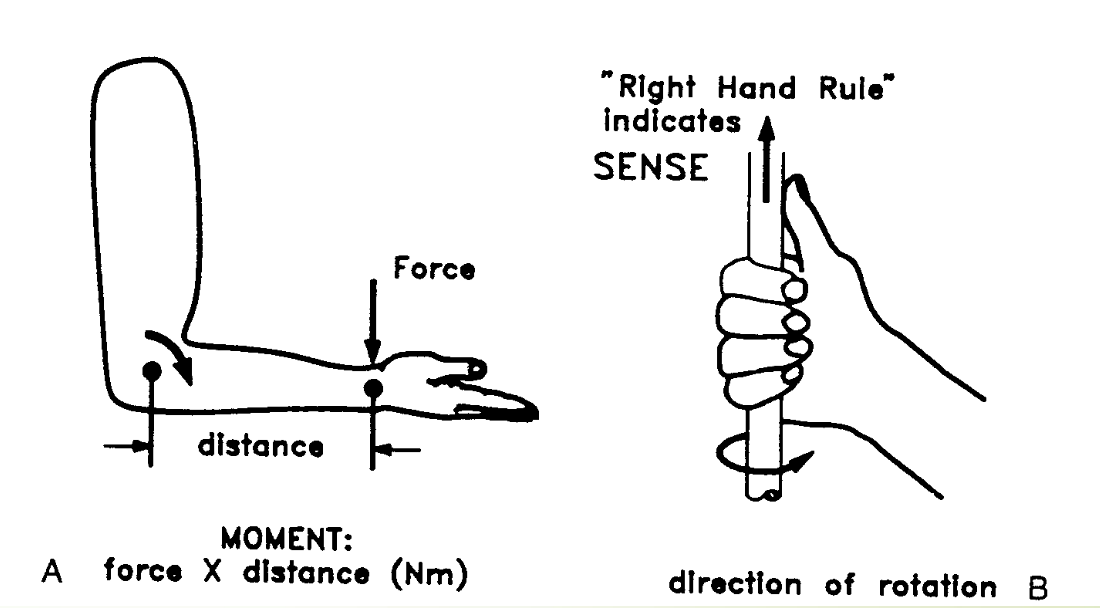
Basic Mechanics

- Moment
  - Moment (M) represents the turning, twisting or rotational effect of a force

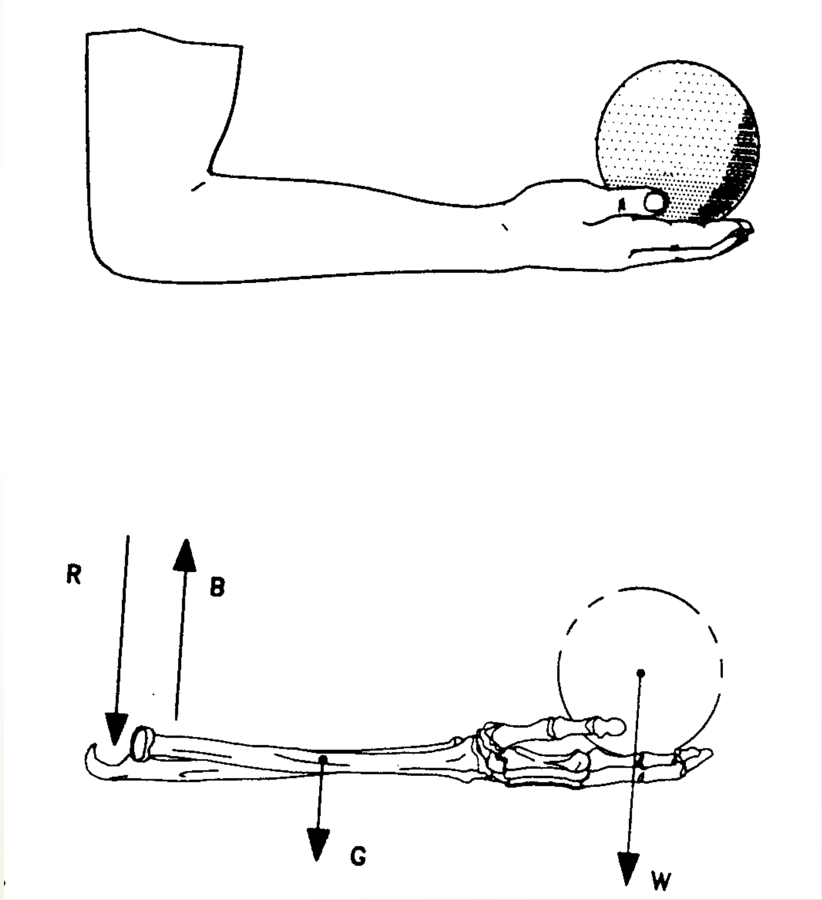


Newton's Laws

- First law
  - an object at rest will remain at rest or an object in motion will move in a straight line with constant velocity if the net force acting on the object is zero.
- Second law
  - an object with a nonzero net force acting on it will accelerate in the direction of the net force and that the magnitude of the acceleration will be proportional to the magnitude of the net force. Newton's second law can be formulated as $\underline{F} = m\,\underline{a}$. ($\underline{M} = I\,\underline{a}$)
- Third law
  - states that to every action there is a reaction and that the force of action and reaction between interacting objects are equal in magnitude, opposite in direction, and have the same line of action. This law has important applications in constructing free-body diagrams.

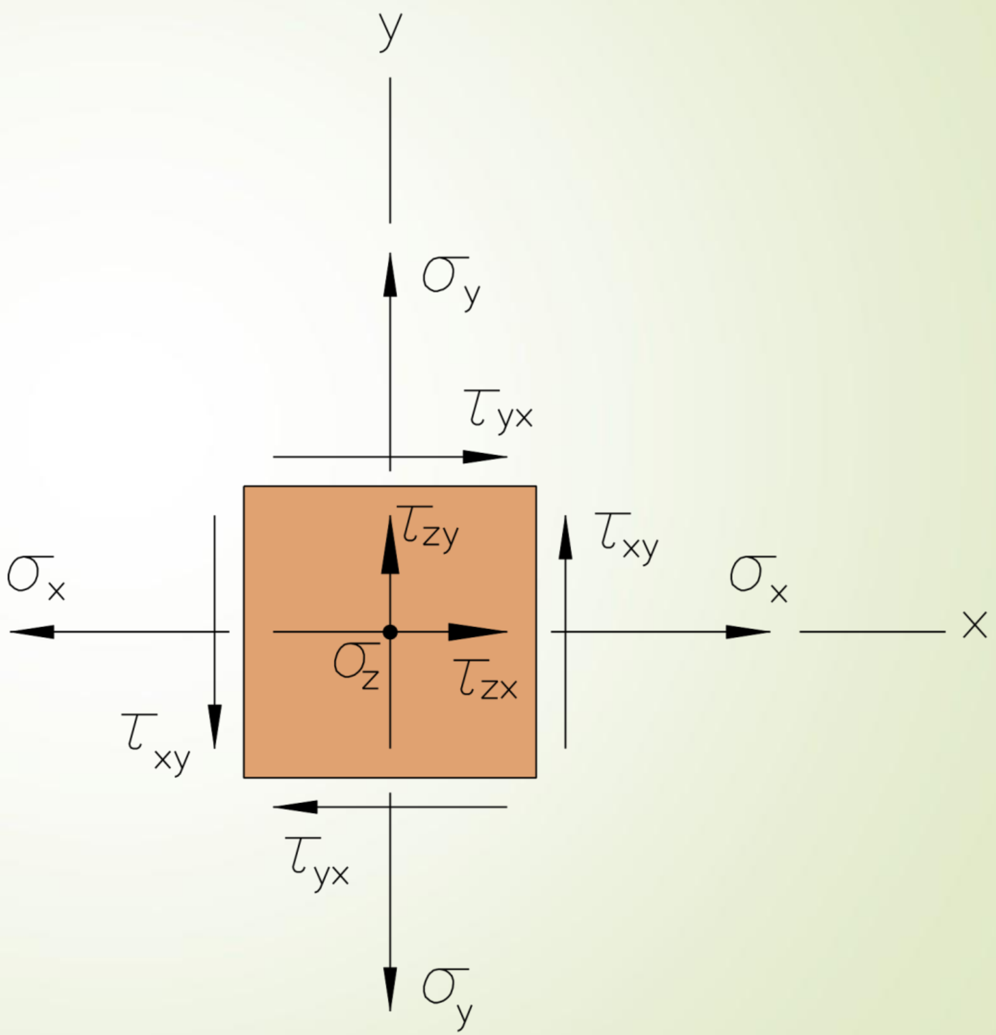
Free-body diagrams

- The diagram of such an isolated body with the representation of all external forces acting on it is called a free-body diagram



Equilibrium

- 2D –Plane case
- Static Equilibrium
  - $\Sigma F = 0; \Sigma M = 0$



Modes of Deformation

- When acted on by externally applied forces, objects may translate in the direction of the net force and rotate in the direction of the net torque acting on them.
- If an object is subjected to externally applied force but is in static equilibrium, then it is most likely that there is some local shape change within the object.
- Local shape change under the effect of applied forces is known as deformation.
- The extent of deformation an object may undergo depends on many factors, including the material properties, size, and shape of the object; environmental factors such as heat and humidity; and the magnitude, direction, and duration of applied forces.
- Mechanics (material)

Fifth SI Edition

CHAPTER

1

MECHANICS OF MATERIALS

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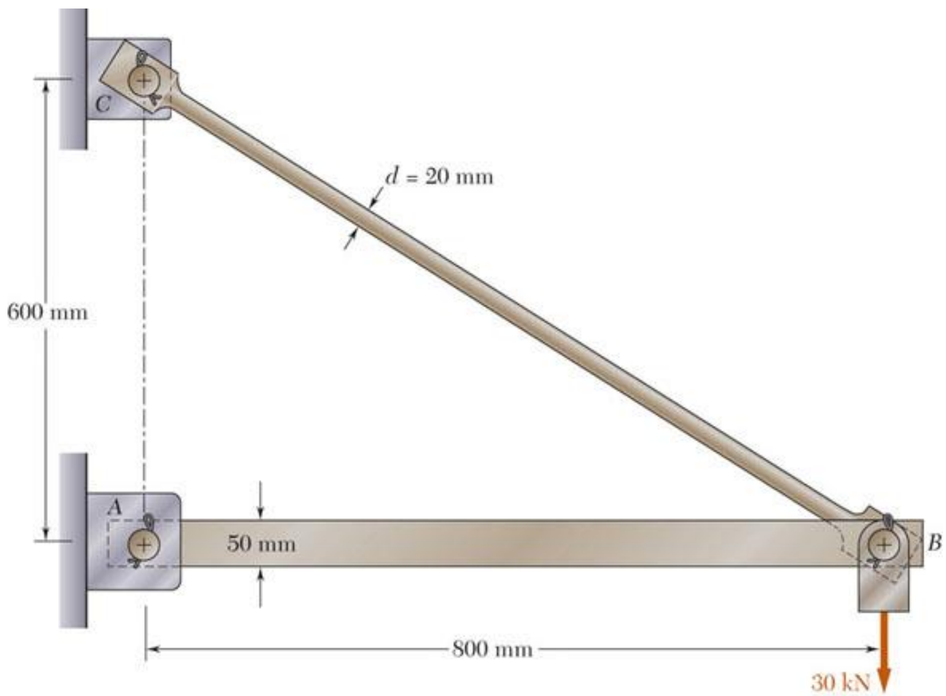
Introduction – Concept of Stress

Concept of Stress

- The main objective of the study of the mechanics of materials is to provide the future engineer with the means of analyzing and designing various machines and load bearing structures.

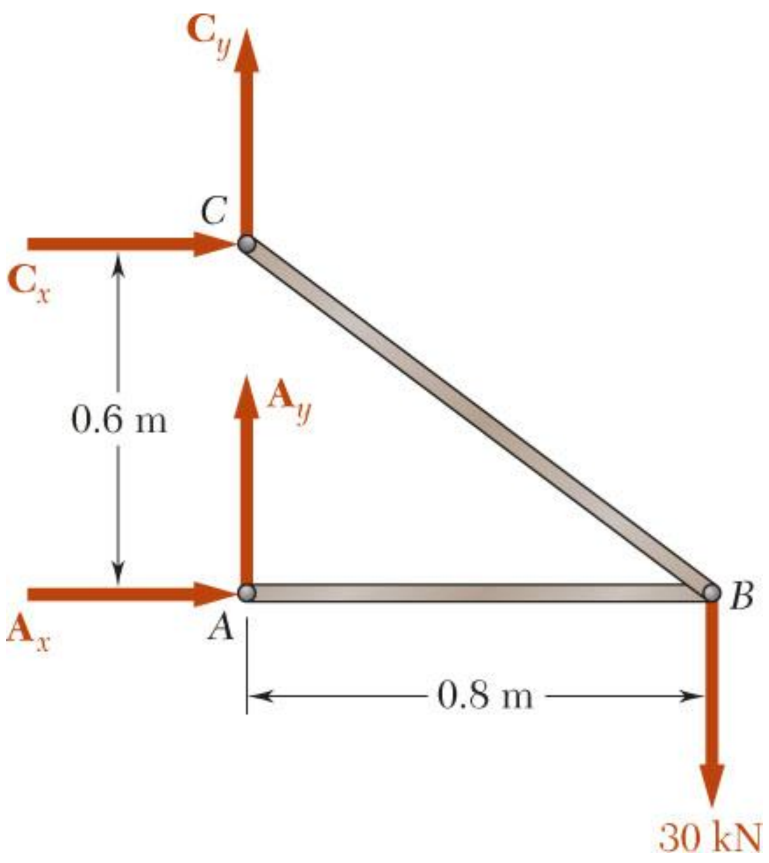
- Both the analysis and design of a given structure involve the determination of *stresses* and *deformations*. This chapter is devoted to the concept of stress.

1.1 Review of Methods of Statics



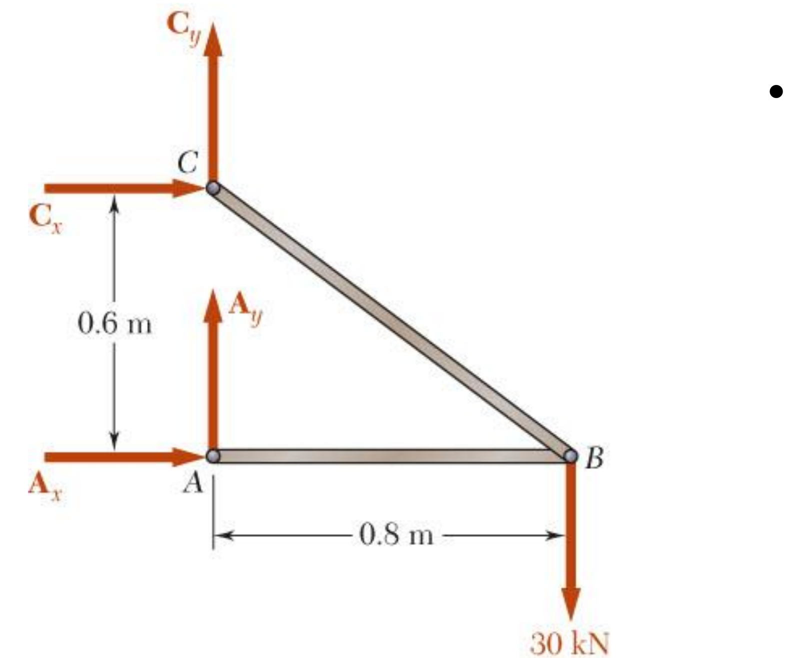
- The structure is designed to support a 30 kN load
- The structure consists of a boom and rod joined by pins (zero moment connections) at the junctions and supports
- Perform a static analysis to determine the internal force in each structural member and the reaction forces at the supports

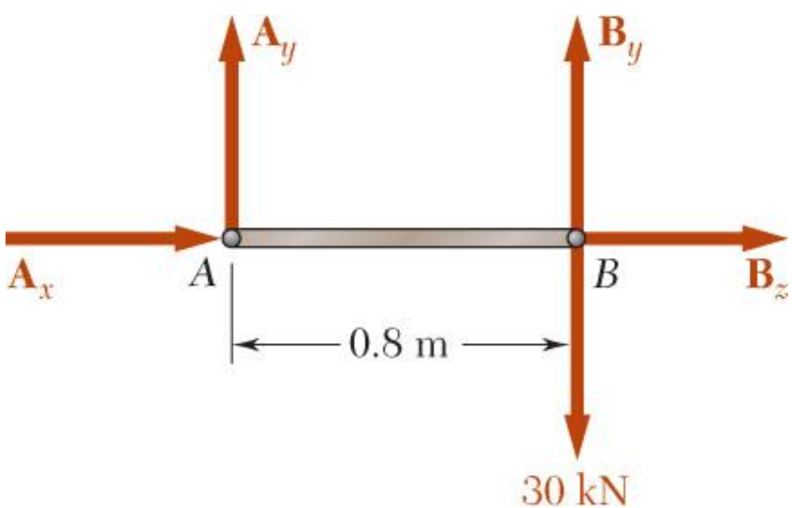
Structure Free-Body Diagram



- Structure is detached from supports and the loads and reaction forces are indicated

Component Free-Body Diagram





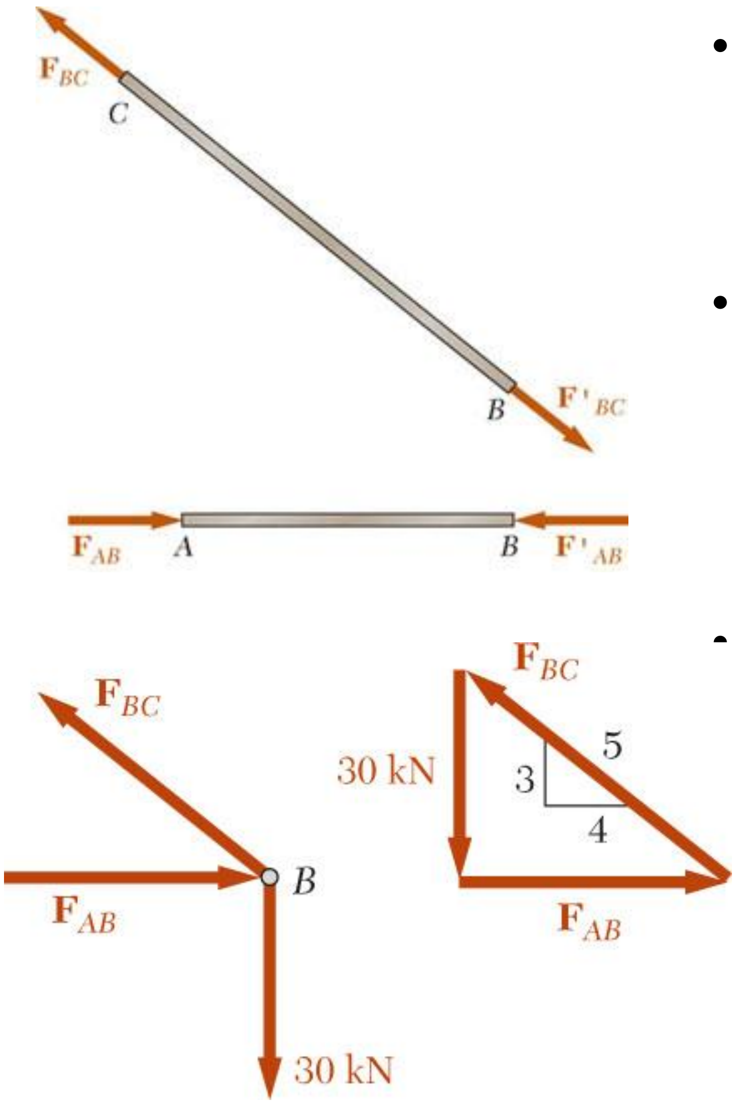
- In addition to the complete structure, each component must satisfy the conditions for static equilibrium

Results:

$$A = 40\,\text{kN} \rightarrow \quad C_x = 40\,\text{kN} \leftarrow \quad C_y = 30\,\text{kN} \uparrow$$

Reaction forces are directed along boom and rod

Method of Joints



- The boom and rod are 2-force members, i.e., the members are subjected to only two forces which are applied at member ends

- For equilibrium, the forces must be parallel to to an axis between the force application points, equal in magnitude, and in opposite directions

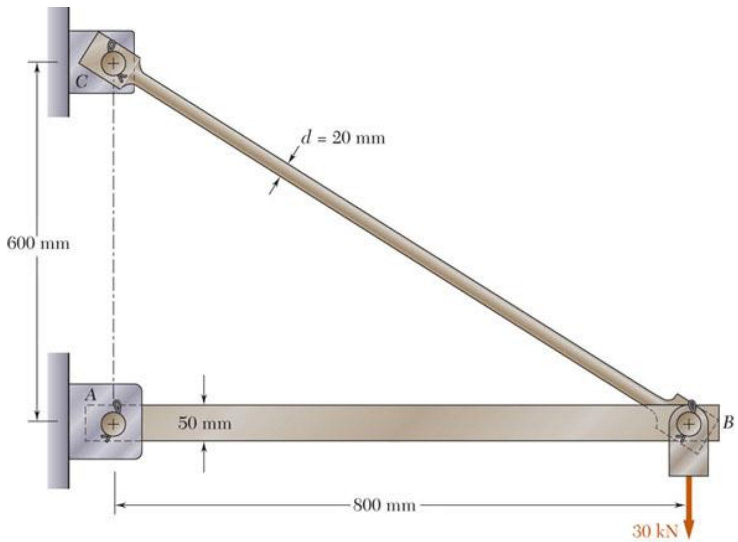
- Joints must satisfy the conditions for static equilibrium which may be expressed in the form of a force triangle:

$$\sum \vec{F}_B = 0$$

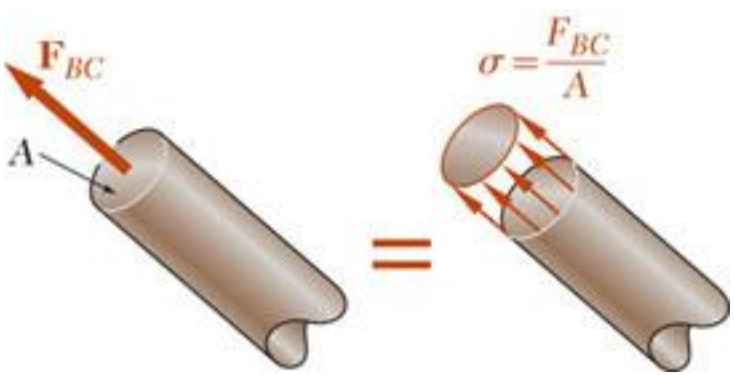
$$\frac{F_{AB}}{4} = \frac{F_{BC}}{5} = \frac{30\,\text{kN}}{3}$$

$$F_{AB} = 40\,\text{kN} \qquad F_{BC} = 50\,\text{kN}$$

1.2 Stress in the members of a structure



$d_{BC} = 20$ mm



Can the structure safely support the 30 kN load?

- From a statics analysis

  $F_{AB} = 40$ kN (compression)
  $F_{BC} = 50$ kN (tension)

- At any section through member BC, the internal force is 50 kN with a force intensity or stress of

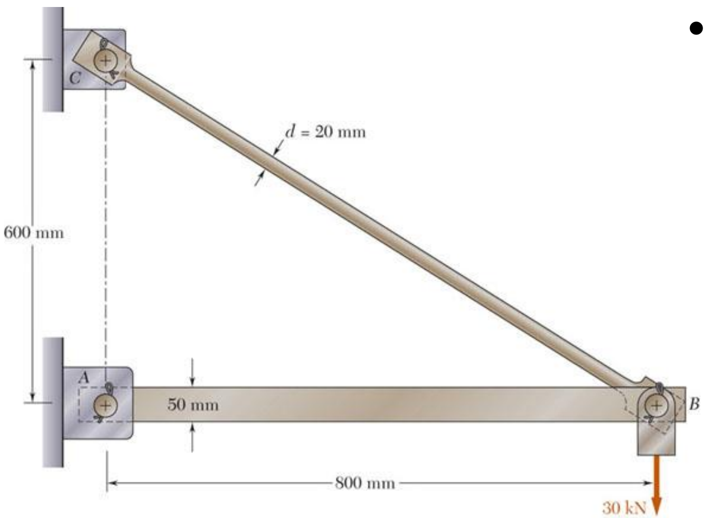
$$\sigma_{BC} = \frac{P}{A} = \frac{50\times10^{3}\,\text{N}}{314\times10^{-6}\,\text{m}^{2}} = 159\ \text{MPa}$$

- From the material properties for steel, the allowable stress is

$$\sigma_{\text{all}} = 165\ \text{MPa}$$

- Conclusion: the strength of member *BC* is adequate

Design



- Design of new structures requires selection of appropriate materials and component dimensions to meet performance requirements

- For reasons based on cost, weight, availability, etc., the choice is made to construct the rod from aluminum (σ_{all}= 100 MPa). What is an appropriate choice for the rod diameter?

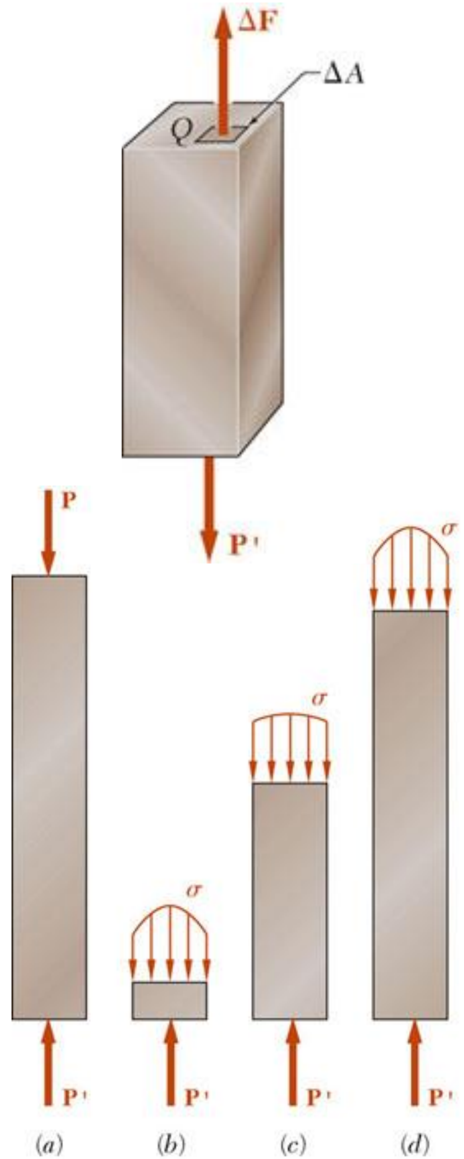
$$\sigma_{all} = \frac{P}{A} \qquad A = \frac{P}{\sigma_{all}} = \frac{50 \times 10^3 \,\mathrm{N}}{100 \times 10^6 \,\mathrm{Pa}} = 500 \times 10^{-6}\,\mathrm{m}^2$$

$$A = \pi \frac{d^2}{4}$$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4\left(500 \times 10^{-6}\,\mathrm{m}^2\right)}{\pi}} = 2.52 \times 10^{-2}\,\mathrm{m} = 25.2\,\mathrm{mm}$$

- An aluminum rod 26 mm or more in diameter is adequate

1.2 A Axial Loading: Normal Stress p8



- The resultant of the internal forces for an axially loaded member is *normal* to a section cut perpendicular to the member axis.

- The force intensity on that section is defined as the normal stress.

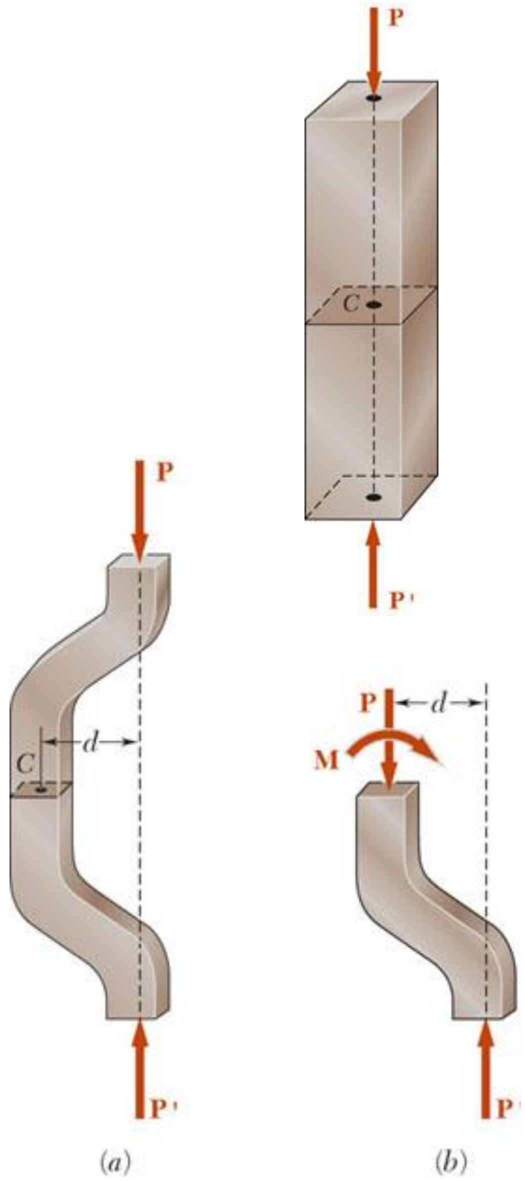
$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} \qquad \sigma_{ave} = \frac{P}{A}$$

- The normal stress at a particular point may not be equal to the average stress but the resultant of the stress distribution must satisfy

$$P = \sigma_{ave} A = \int dF = \int_A \sigma \, dA$$

- The detailed distribution of stress is statically indeterminate, i.e., can not be found from statics alone.

Centric & Eccentric Loading



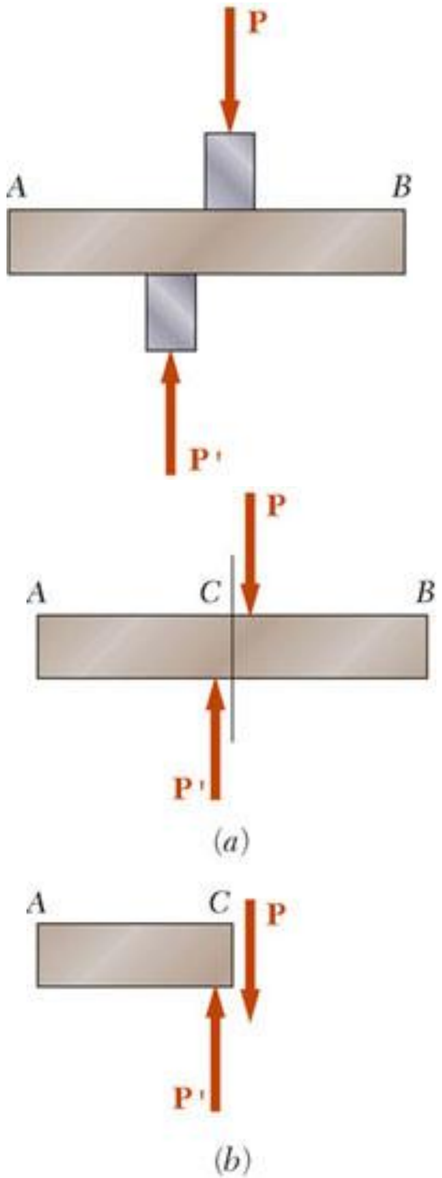
- A uniform distribution of stress in a section infers that the line of action for the resultant of the internal forces passes through the centroid of the section.

- A uniform distribution of stress is only possible if the concentrated loads on the end sections of two-force members are applied at the section centroids. This is referred to as *centric loading*.

- If a two-force member is *eccentrically loaded*, then the resultant of the stress distribution in a section must yield an axial force and a moment.

- The stress distributions in eccentrically loaded members cannot be uniform or symmetric.

1.2 B Shearing Stress



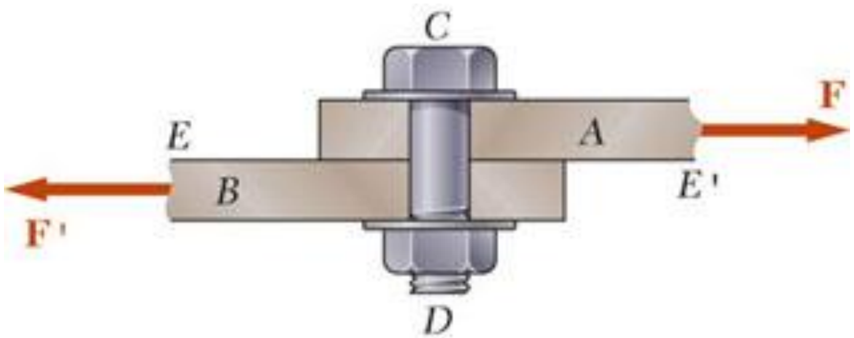
- Forces *P* and *P'* are applied transversely to the member *AB*.
- Corresponding internal forces act in the plane of section *C* and are called *shearing* forces.
- The resultant of the internal shear force distribution is defined as the *shear* of the section and is equal to the load *P*.
- The corresponding average shear stress is,

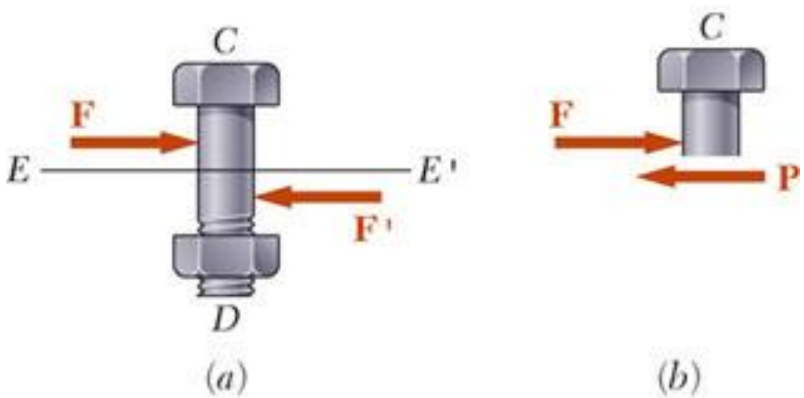
$$\tau_{\text{ave}} = \frac{P}{A}$$

- Shear stress distribution varies from zero at the member surfaces to maximum values that may be much larger than the average value.
- The shear stress distribution cannot be assumed to be uniform.

Shearing Stress Examples

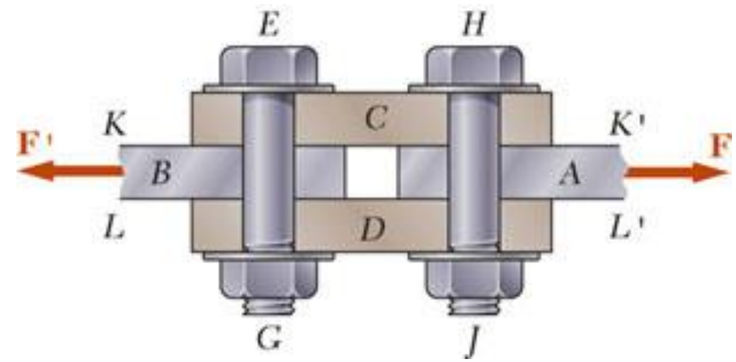
Single Shear

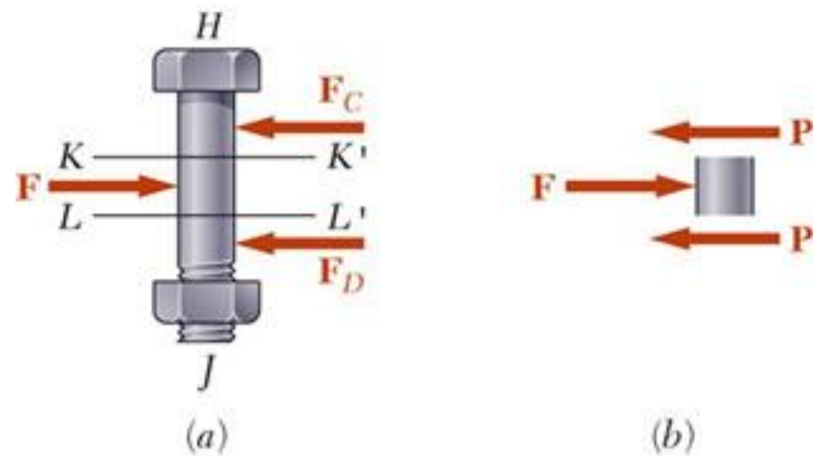




$$\tau_{\text{ave}} = \frac{P}{A} = \frac{F}{A}$$

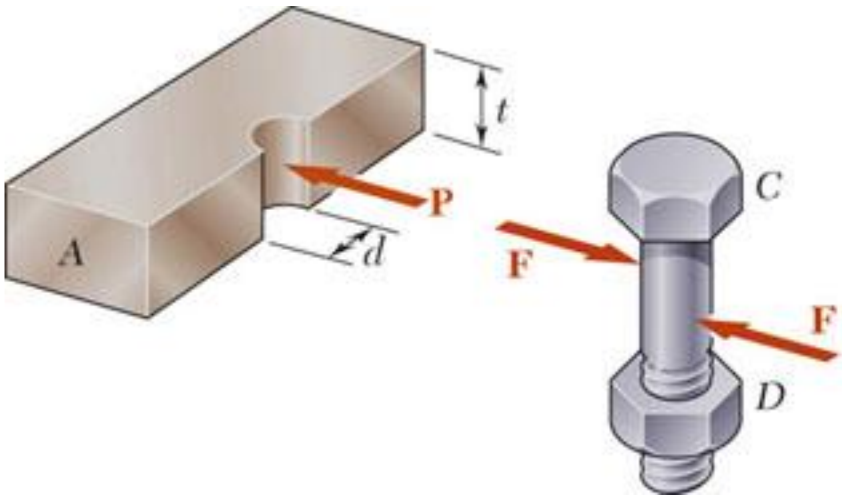
Double Shear

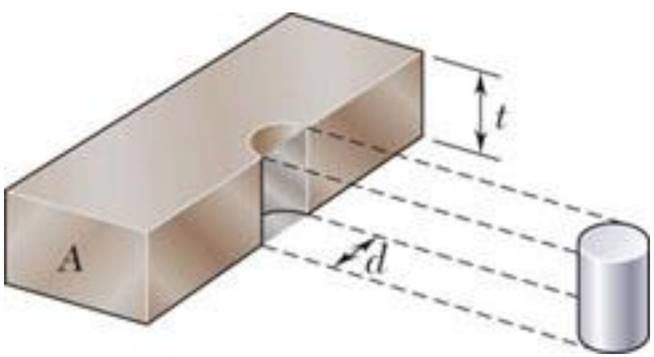




$$\tau_{\text{ave}} = \frac{P}{A} = \frac{F}{2A}$$

1.2 C Bearing Stress in Connections

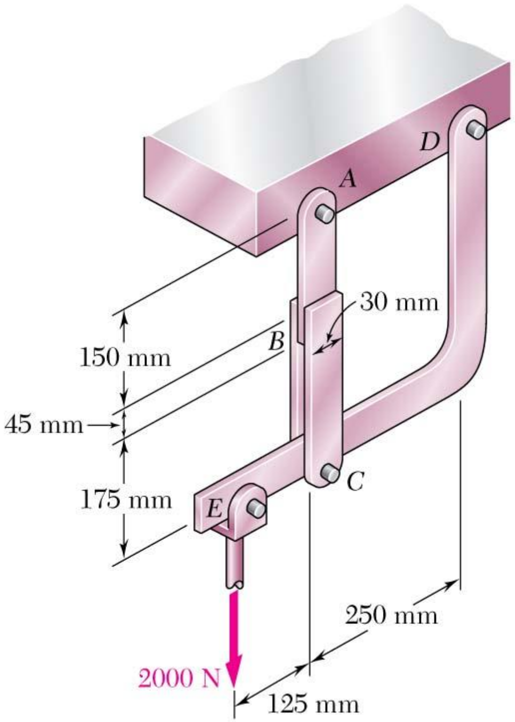


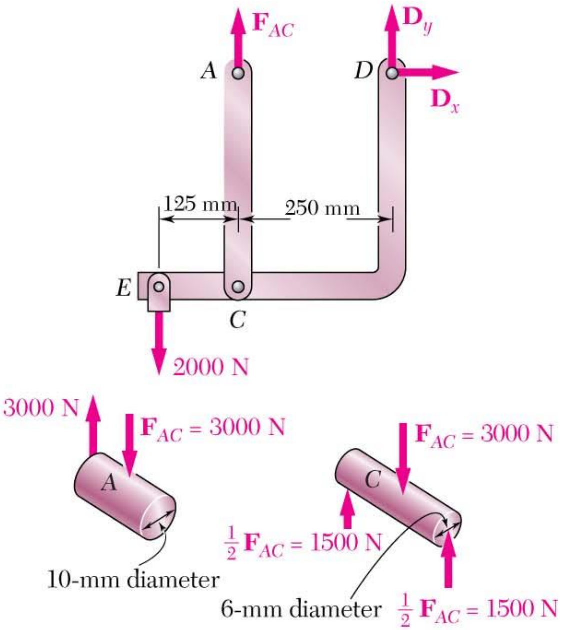


- Bolts, rivets, and pins create stresses on the points of contact or *bearing surfaces* of the members they connect.
- The resultant of the force distribution on the surface is equal and opposite to the force exerted on the pin.
- Corresponding average force intensity is called the bearing stress,

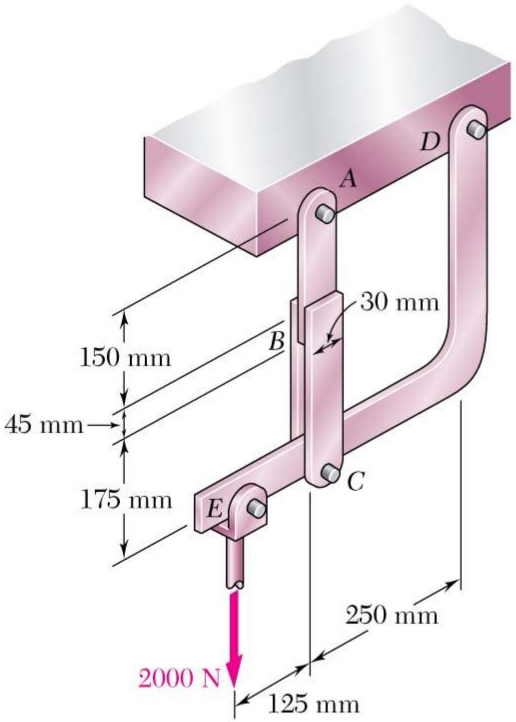
$$\sigma_b = \frac{P}{A} = \frac{P}{t\,d}$$

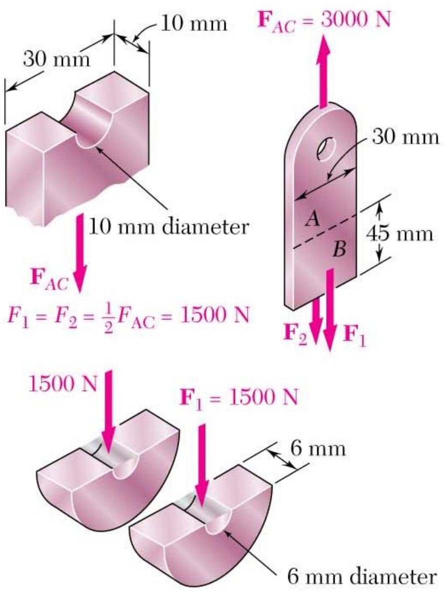
Sample Problems 1.1



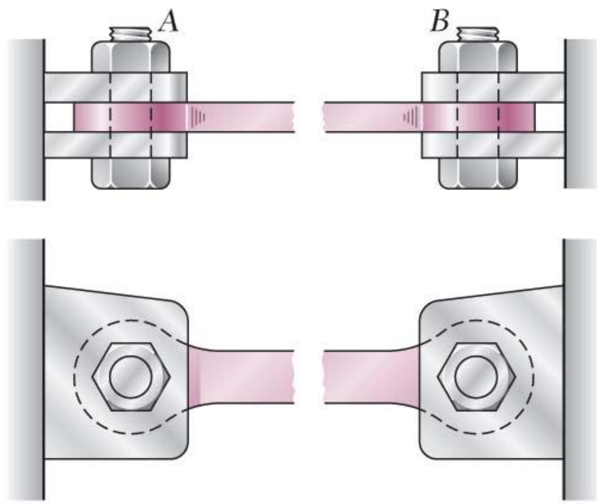


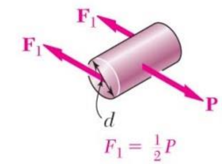
Sample Problem 1.1

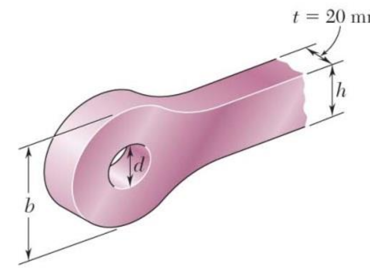


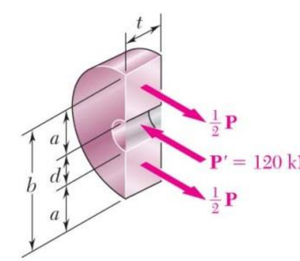


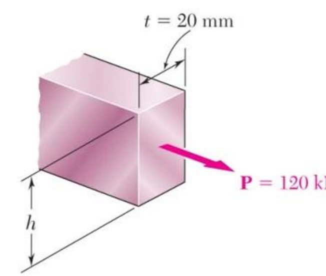
Sample Problem 1.2 p19











1.5 C Factor of Safety Selection p33

Structural members or machines must be designed such that the working stresses are less than the ultimate strength of the material.

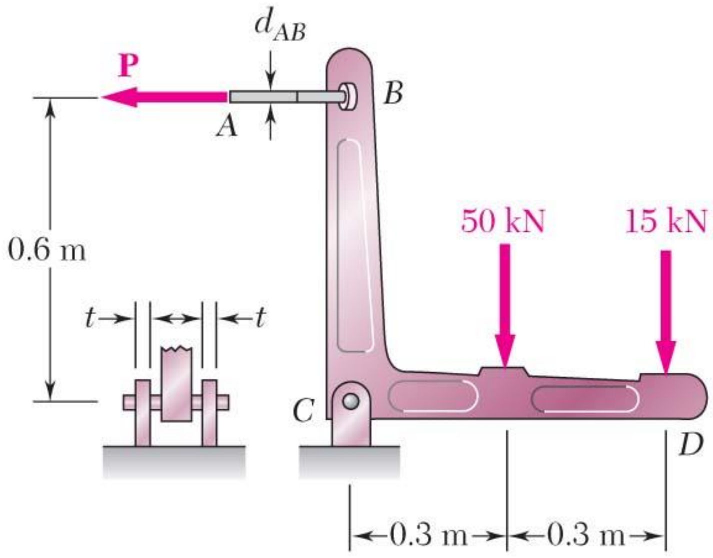
$$FS = \text{Factor of safety}$$

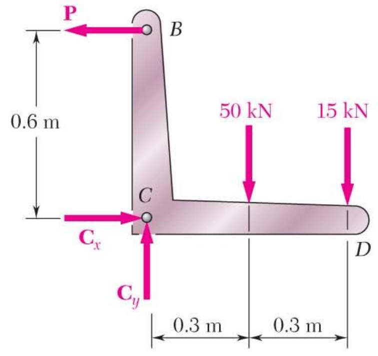
$$FS = \frac{\sigma_u}{\sigma_{all}} = \frac{\text{ultimate stress}}{\text{allowable stress}}$$

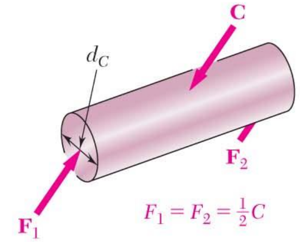
Factor of safety considerations:

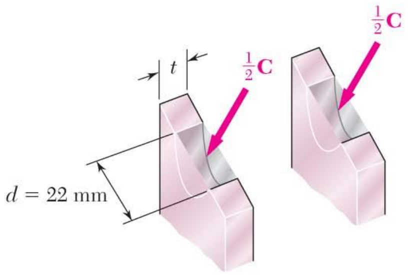
- uncertainty in material properties
- uncertainty of loadings
- uncertainty of analyses
- number of loading cycles
- types of failure
- maintenance requirements and deterioration effects
- importance of member to integrity of whole structure
- risk to life and property
- influence on machine function

Sample Problem 1.3 p35

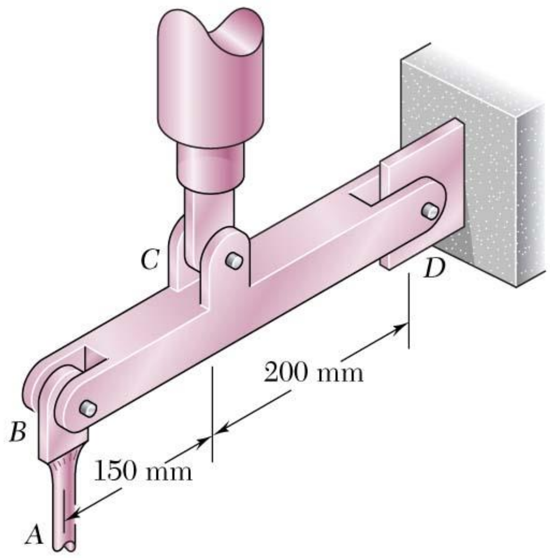


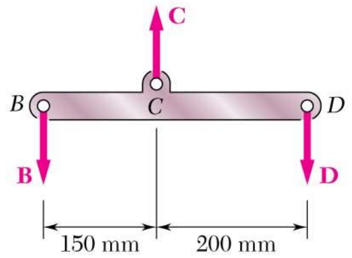


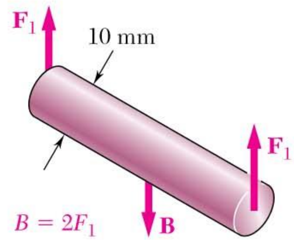


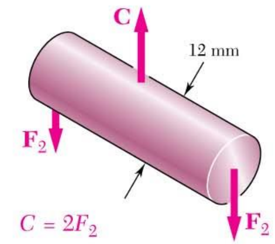


Sample Problem 1.4 p36









Problems

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