CHAPTER

2

MECHANICS OF MATERIALS

Ferdinand P. Beer
E. Russell Johnston, Jr.
John T. DeWolf
David F. Mazurek

Lecture Notes:
J. Walt Oler
Texas Tech University

Stress and Strain – Axial Loading

2.1 An Introduction to Stress and Strain
2.1 A Normal Strain under Axial Loading

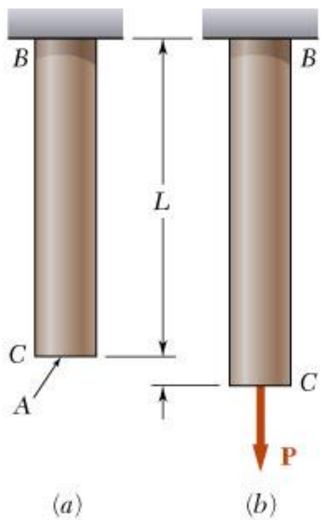


Fig. 2.1

$$\sigma = \frac{P}{A} = \text{stress}$$

$$\varepsilon = \frac{\delta}{L} = \text{normal strain}$$

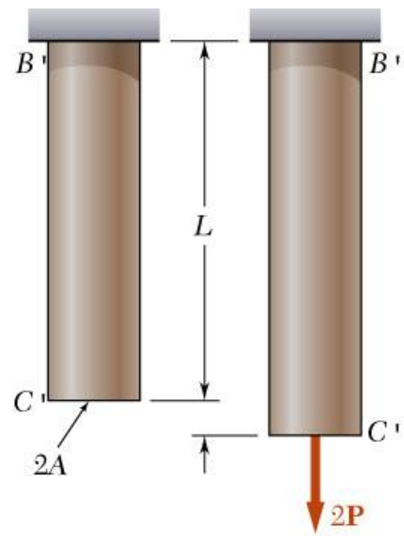


Fig. 2.3

$$\sigma = \frac{2P}{2A} = \frac{P}{A}$$

$$\varepsilon = \frac{\delta}{L}$$

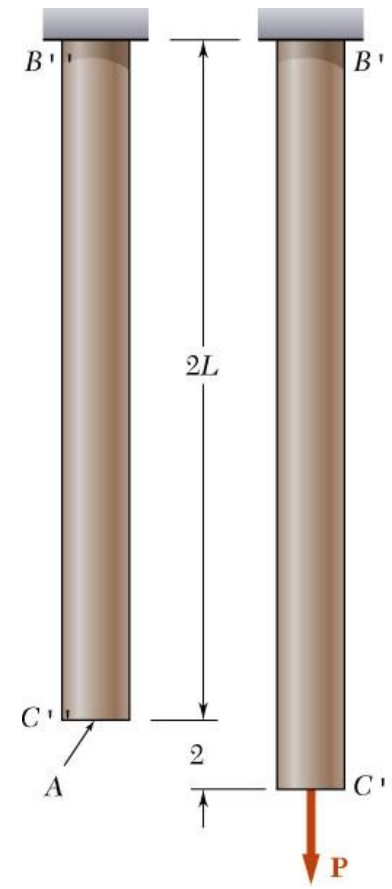
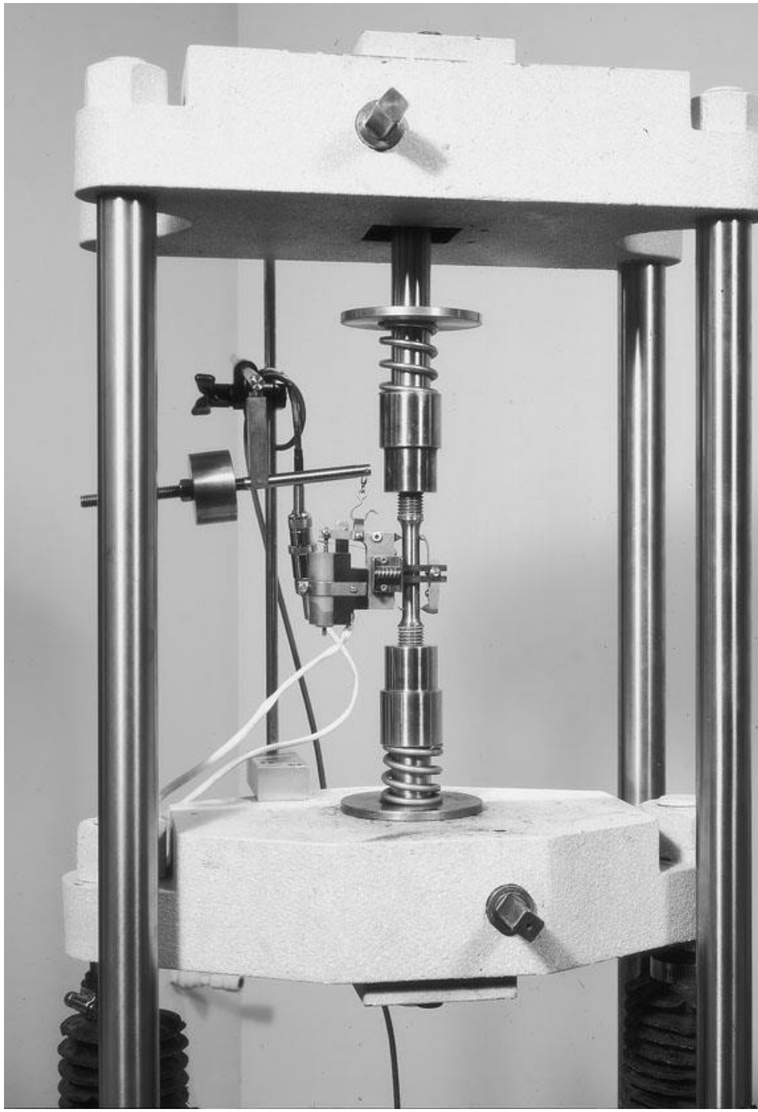


Fig. 2.4

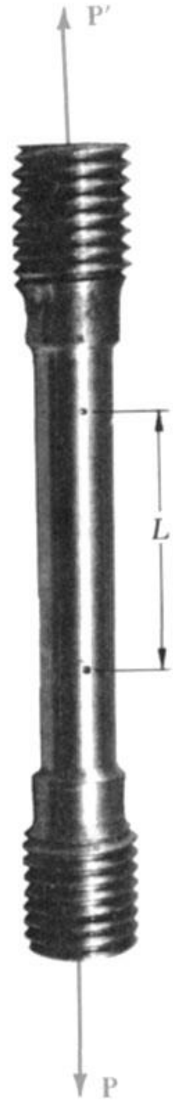
$$\sigma = \frac{P}{A}$$

$$\varepsilon = \frac{2\delta}{2L} = \frac{\delta}{L}$$

Stress-Strain Test (Extensometer)



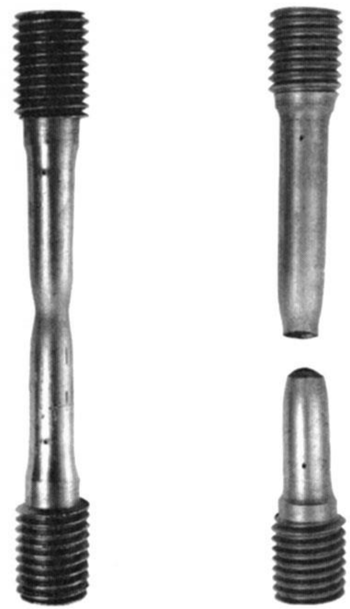
This machine is used to test tensile test specimens, such as those shown in this chapter.



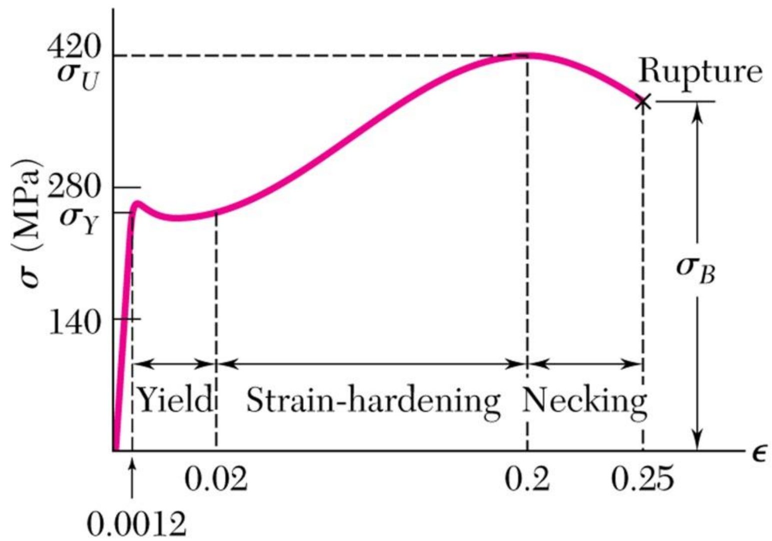
Test specimen with tensile load.

Photo 2.2 Universal test machine used to test tensile specimens.

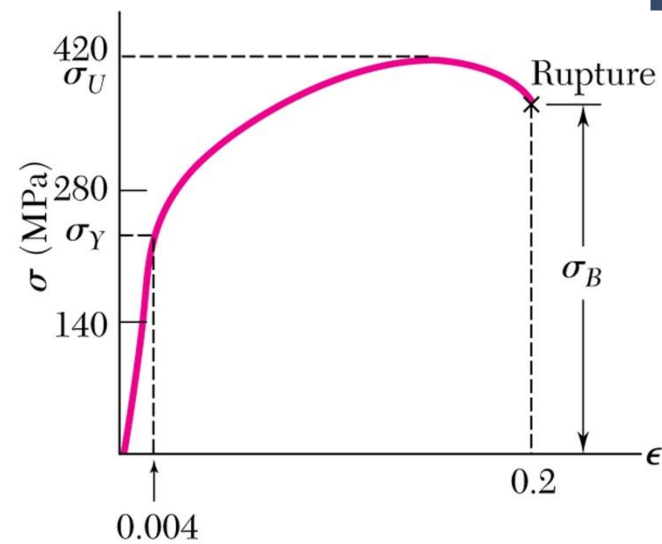
2.1 B Stress-Strain Diagram: Ductile Materials p60



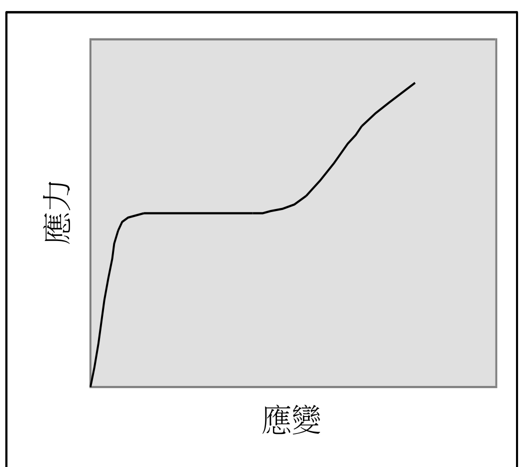
(a) (b)



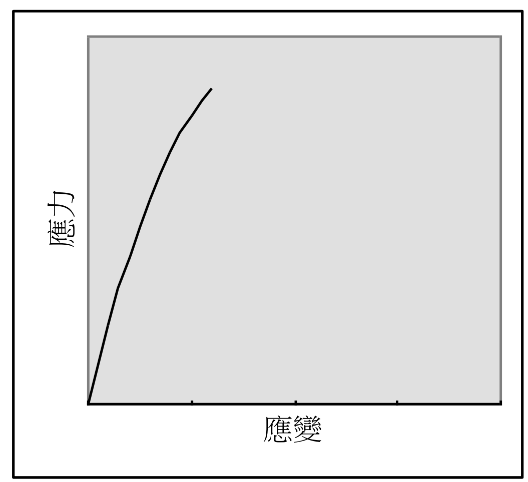
(a) Low-carbon steel



(b) Aluminum alloy

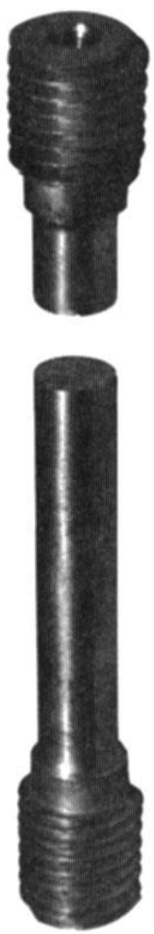


韌性材料



脆性材料

Stress-Strain Diagram: Brittle Materials p61



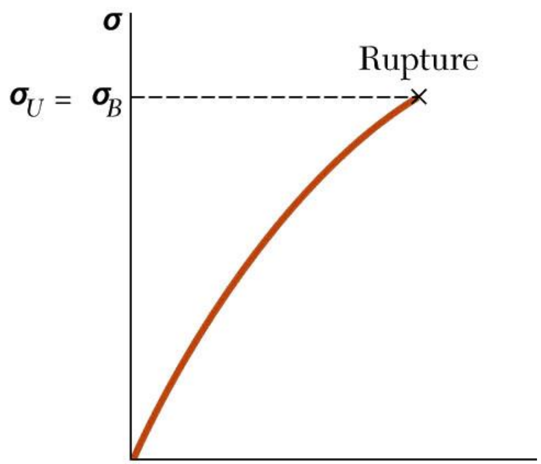


Fig 2.7 Stress-strain diagram for a typical brittle material.

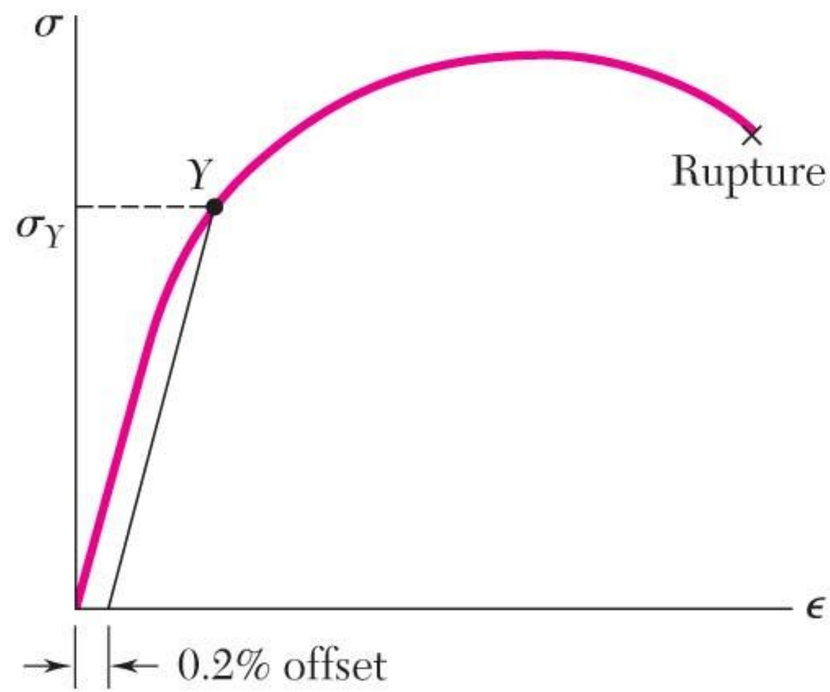


Fig. 2.8 Determination of yield strength by offset method.

2.1 D Hooke's Law: Modulus of Elasticity p63

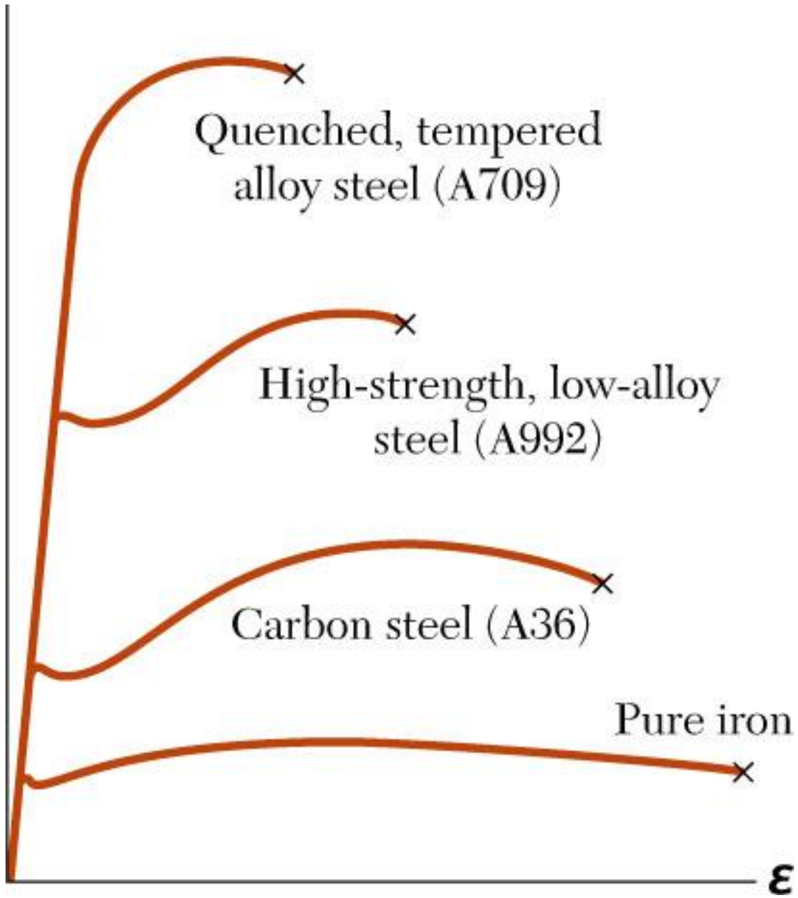


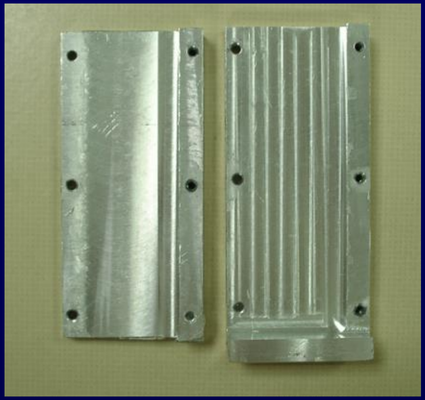
Fig 2.16 Stress-strain diagrams for iron and different grades of steel.

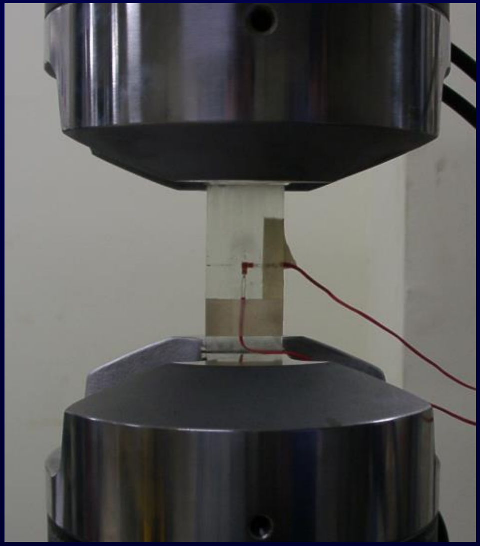
- Below the yield stress

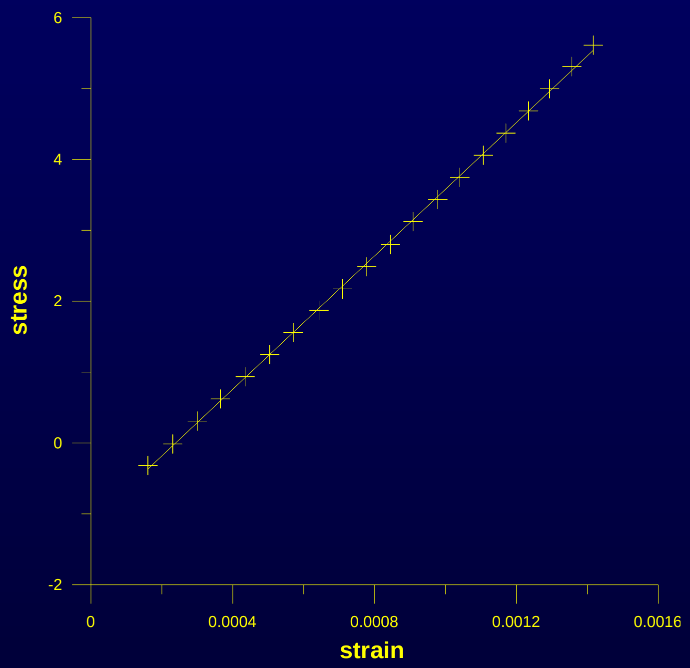
$$\sigma = E\varepsilon$$
$$E = \text{Youngs Modulus or Modulus of Elasticity}$$

- Strength is affected by alloying, heat treating, and manufacturing process but stiffness (Modulus of Elasticity) is not.

Stress-strain behavior







Curve fitting:

Y = 4702.74 X - 1.12 1.12

2.1 E Elastic vs. Plastic Behavior p65

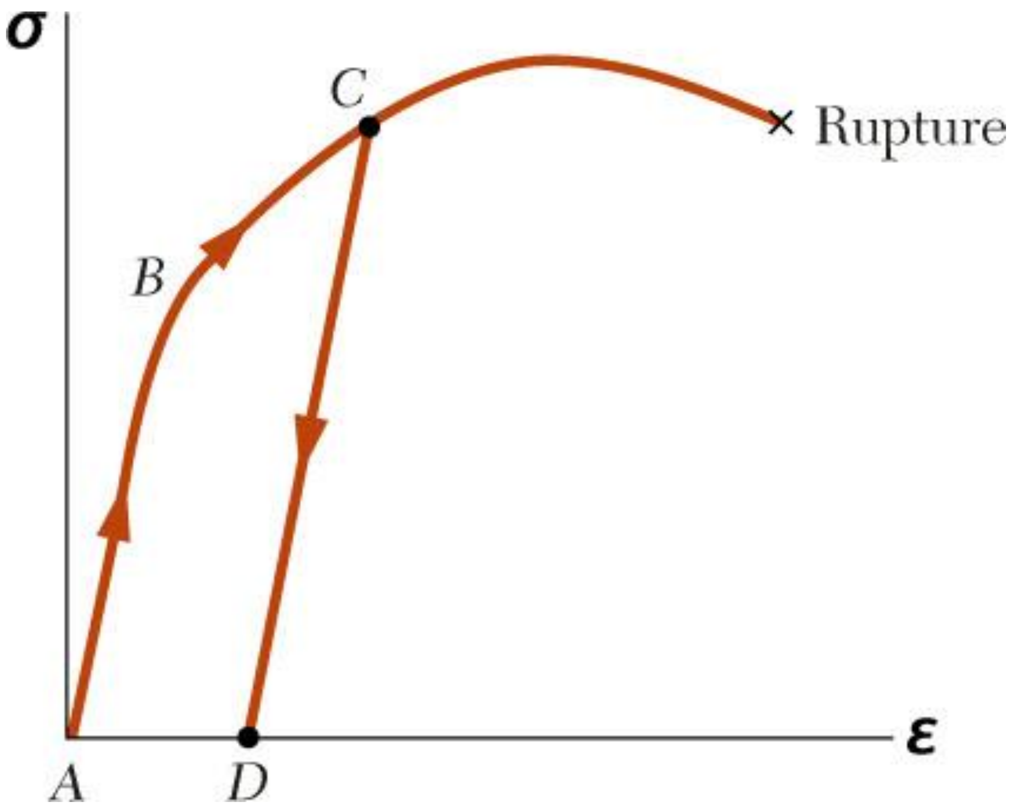


Fig. 2.13

- If the strain disappears when the stress is removed, the material is said to behave *elastically*.
- The largest stress for which this occurs is called the *elastic limit*.
- When the strain does not return to zero after the stress is removed, the material is said to behave *plastically*.

Homogeneous & Isotropic

- **Homogeneous (均質)**
  - 材料的每一個質點都具有相同的材料特性
- **Isotropic (等向)**
  - 材料的性質在每一個方向都一樣

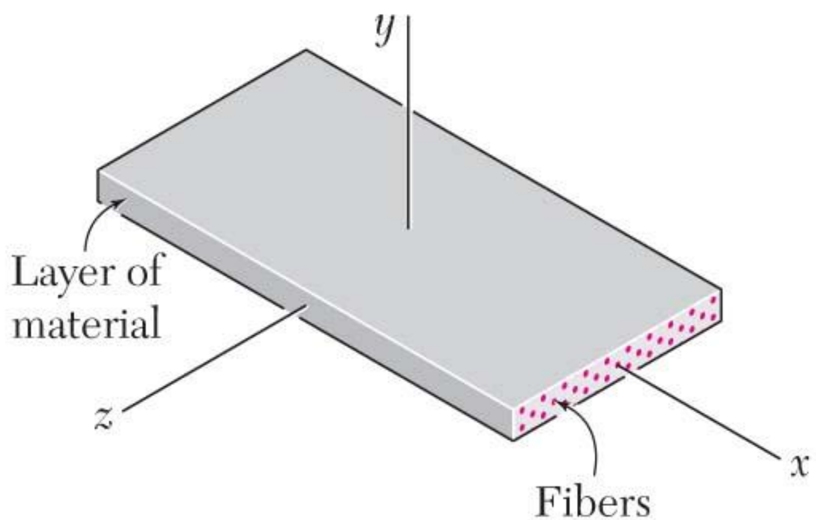


Fig. 2.12 Layer of fiber-reinforced composite material.

2.1 F Repeated Loading and Fatigue

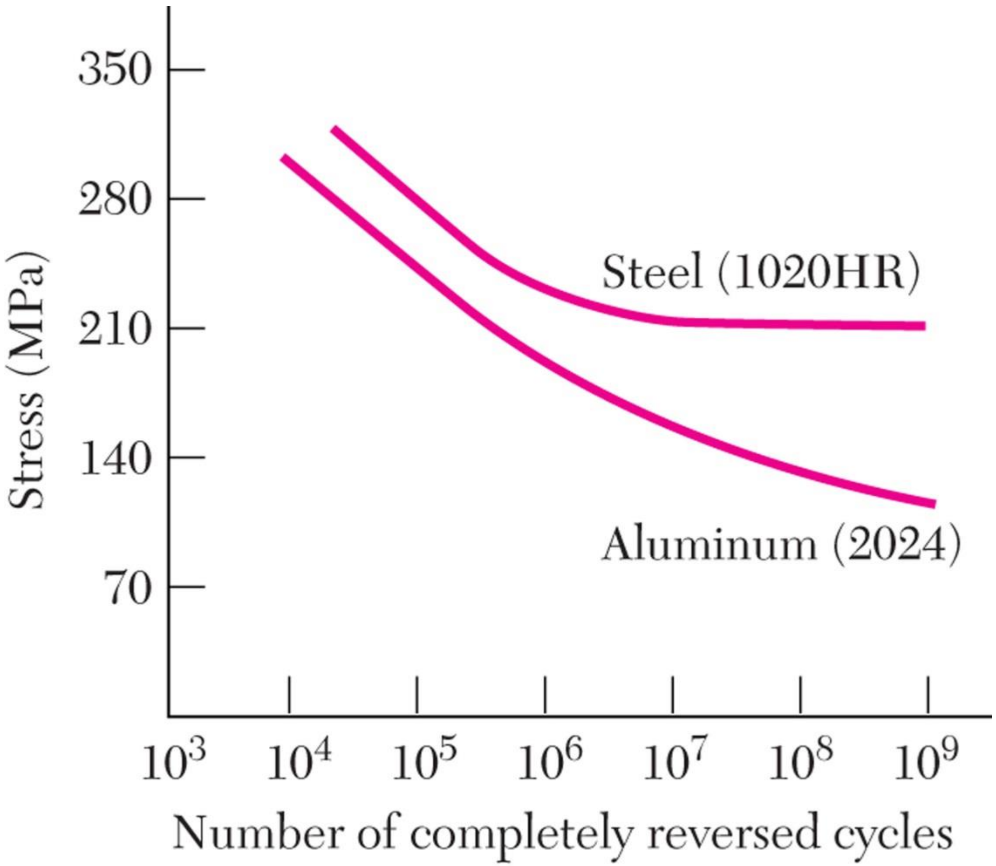


Fig. 2.21

- Fatigue properties are shown on S-N diagrams.
- A member may fail due to *fatigue* at stress levels significantly below the ultimate strength if subjected to many loading cycles.
- When the stress is reduced below the *endurance limit*, fatigue failures do not occur for any number of cycles.

2.1 G Deformations Under Axial Loading p68

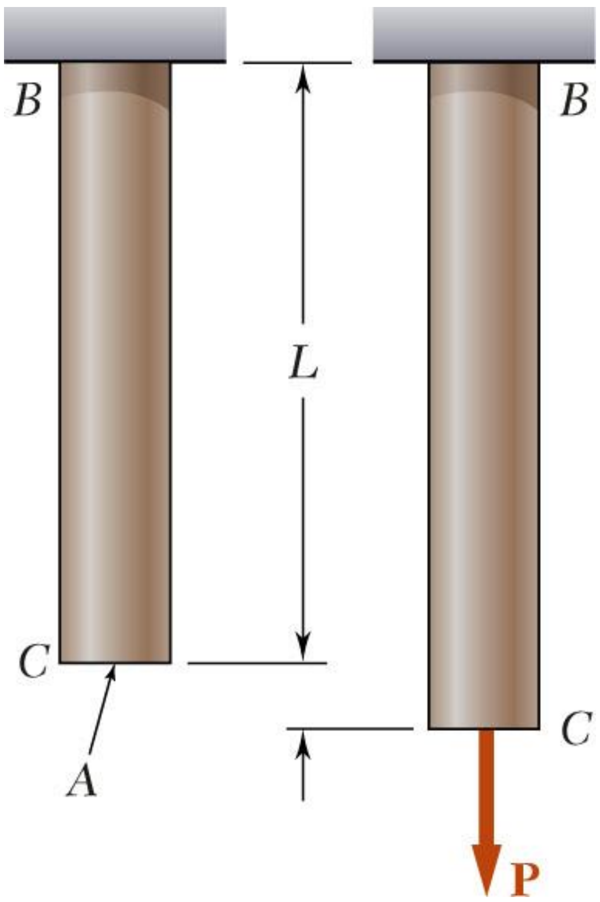


Fig. 2.17

- From Hooke's Law:

$$\sigma = E\varepsilon \qquad \varepsilon = \frac{\sigma}{E} = \frac{P}{AE}$$

- From the definition of strain:

$$\varepsilon = \frac{\delta}{L}$$

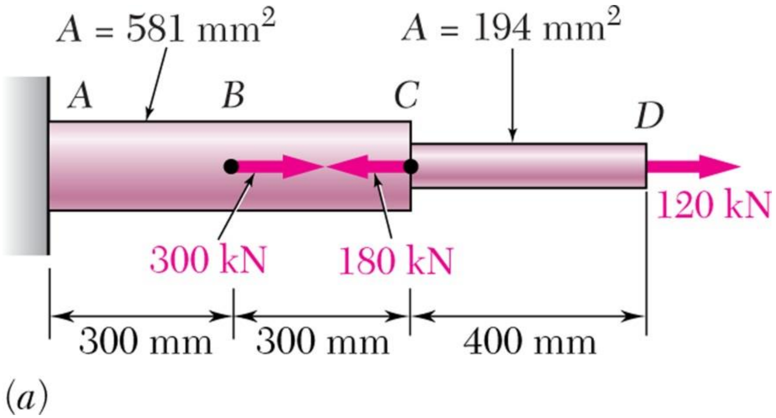
- Equating and solving for the deformation,

$$\delta = \frac{PL}{AE}$$

- With variations in loading, cross-section or material properties,

$$\delta = \sum_i \frac{P_i L_i}{A_i E_i}$$

Concept Application 2.1



$E = 200$ GPa

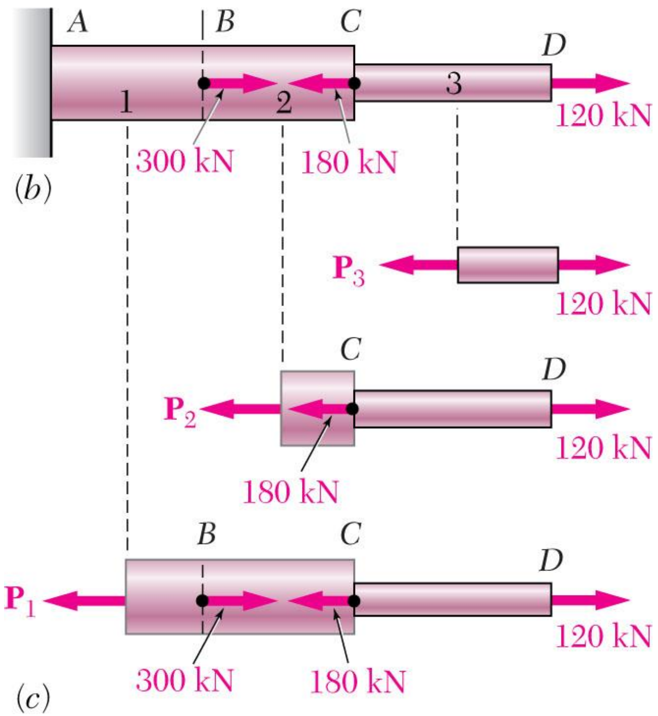
Determine the deformation of the steel rod shown under the given loads.

SOLUTION:

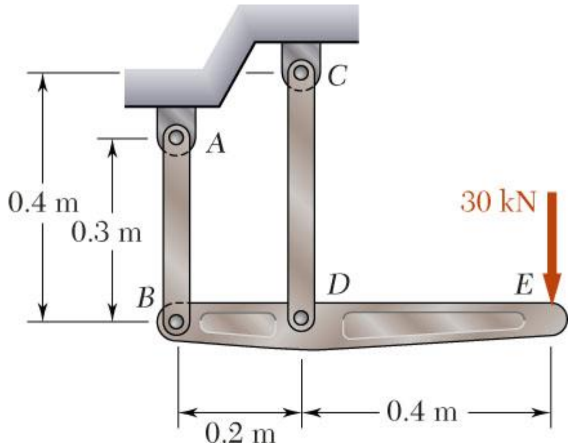
- Divide the rod into components at the load application points.
- Apply a free-body analysis on each component to determine the internal force
- Evaluate the total of the component deflections.

SOLUTION:

- Divide the rod into three components:



Sample Problem 2.1 p70



The rigid bar *BDE* is supported by two links *AB* and *CD*.

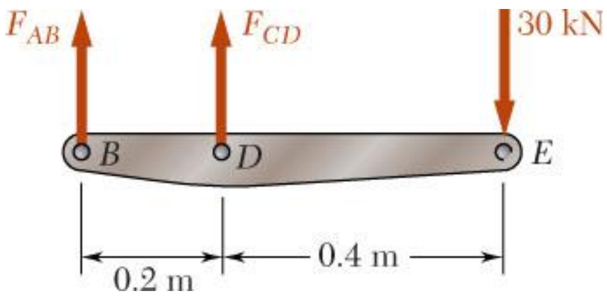
Link *AB* is made of aluminum (E = 70 GPa) and has a cross-sectional area of 500 mm^2. Link *CD* is made of steel (E = 200 GPa) and has a cross-sectional area of (600 mm^2).

For the 30-kN force shown, determine the deflection a) of *B*, b) of *D*, and c) of *E*.

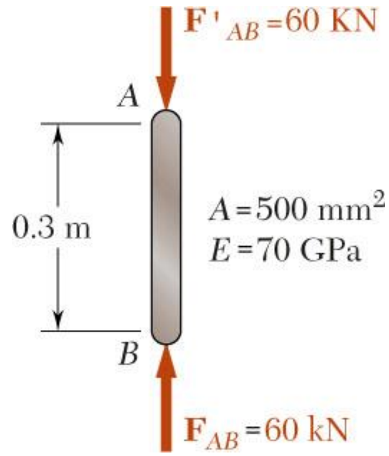
Sample Problem 2.1

SOLUTION:

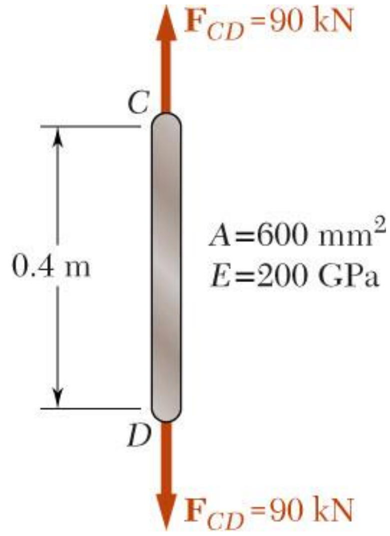
Free body: Bar *BDE*



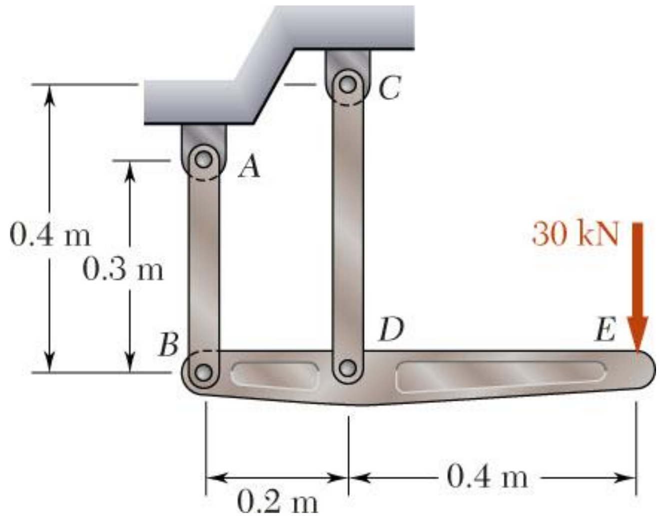
Displacement of *B*:

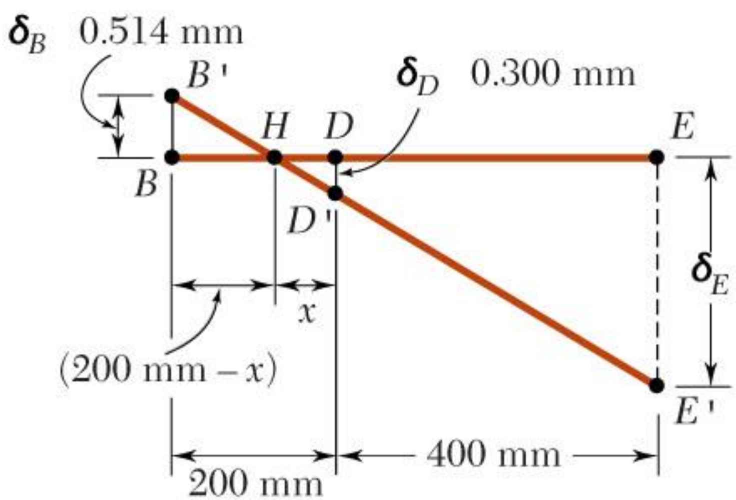


Displacement of *D*:



Sample Problem 2.1





Problems

- Page 71 Sample problem 2.2
- Page 77 2.26, 2.27

2.2 Static Indeterminacy

- Structures for which internal forces and reactions cannot be determined from statics alone are said to be *statically indeterminate*.

- A structure will be statically indeterminate whenever it is held by more supports than are required to maintain its equilibrium.

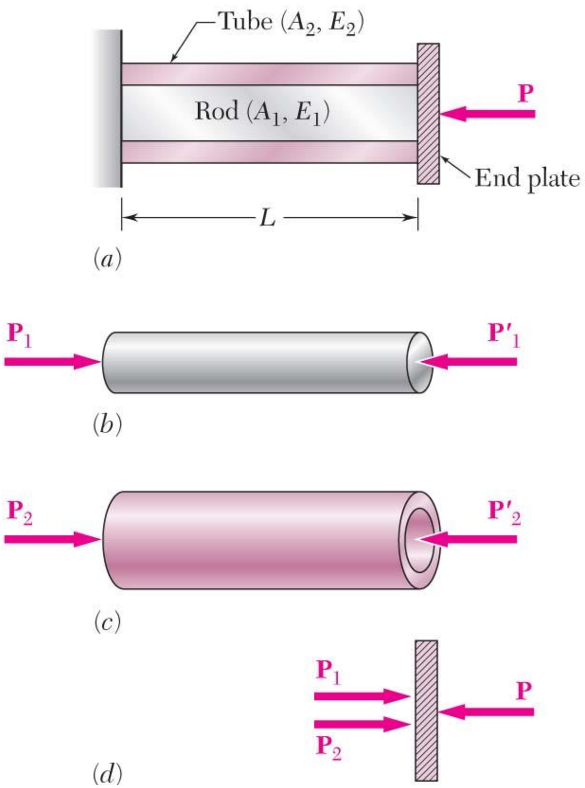
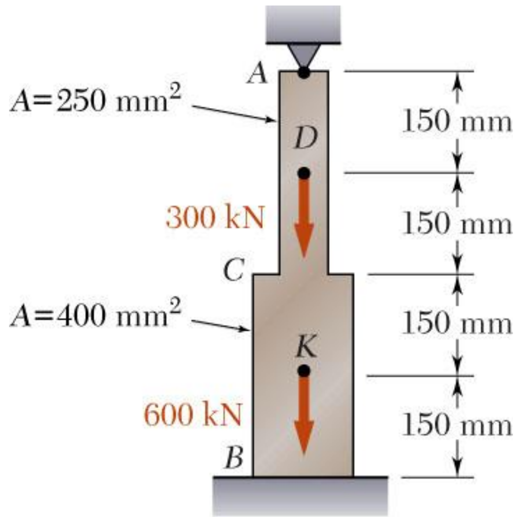
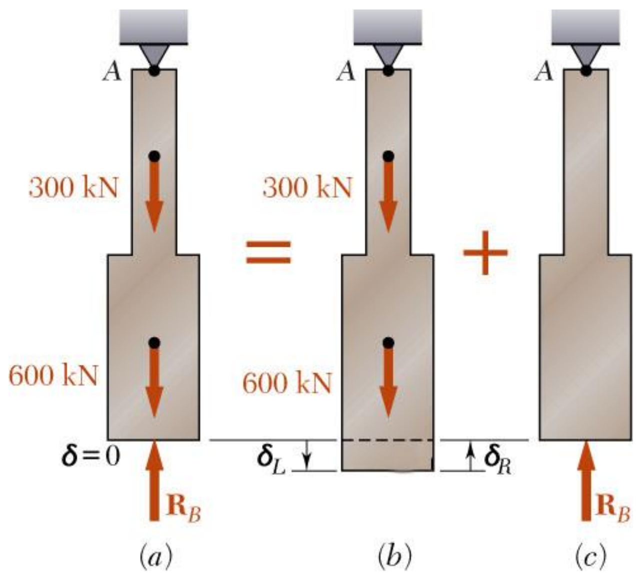


Fig. 2.21

Concept Application 2.4



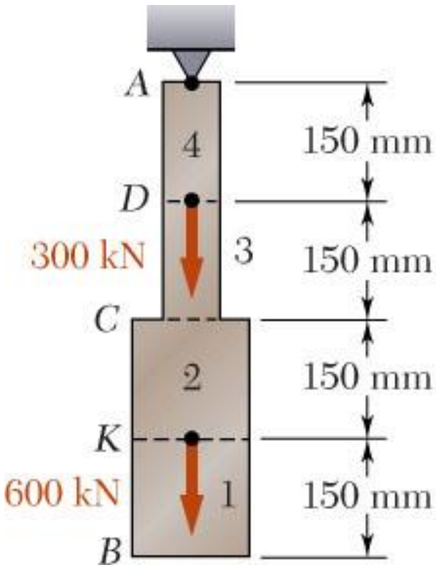


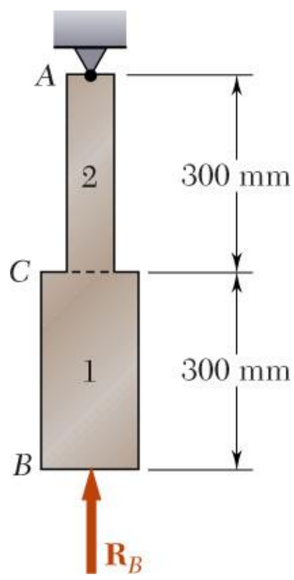
Determine the reactions at *A* and *B* for the steel bar and loading shown, assuming a close fit at both supports before the loads are applied.

SOLUTION:

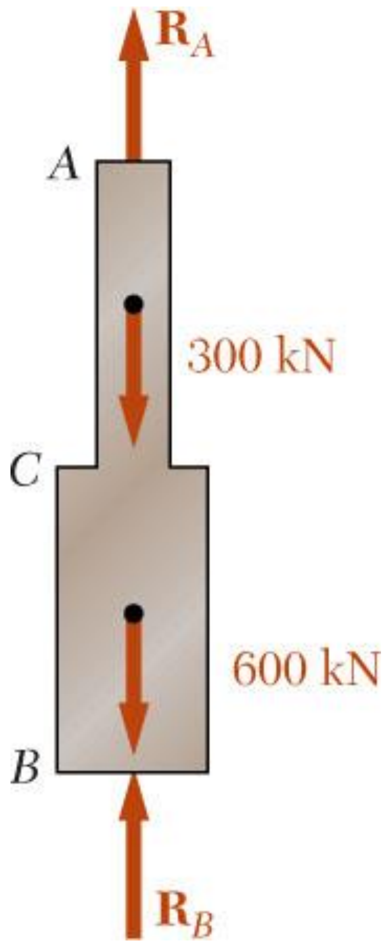
- Consider the reaction at *B* as redundant, release the bar from that support, and solve for the displacement at *B* due to the applied loads.
- Solve for the displacement at *B* due to the redundant reaction at *B.*
- Require that the displacements due to the loads and due to the redundant reaction be compatible, i.e., require that their sum be zero.
- Solve for the reaction at *A* due to applied loads and the reaction found at *B.*

Example 2.04


A
4
150 mm
D
300 kN
3
150 mm
C
2
150 mm
K
600 kN
1
150 mm
B


A
2
300 mm
C
1
300 mm
B
R_B

Concept Application 2.04



2.3 Problems involving temperature changes

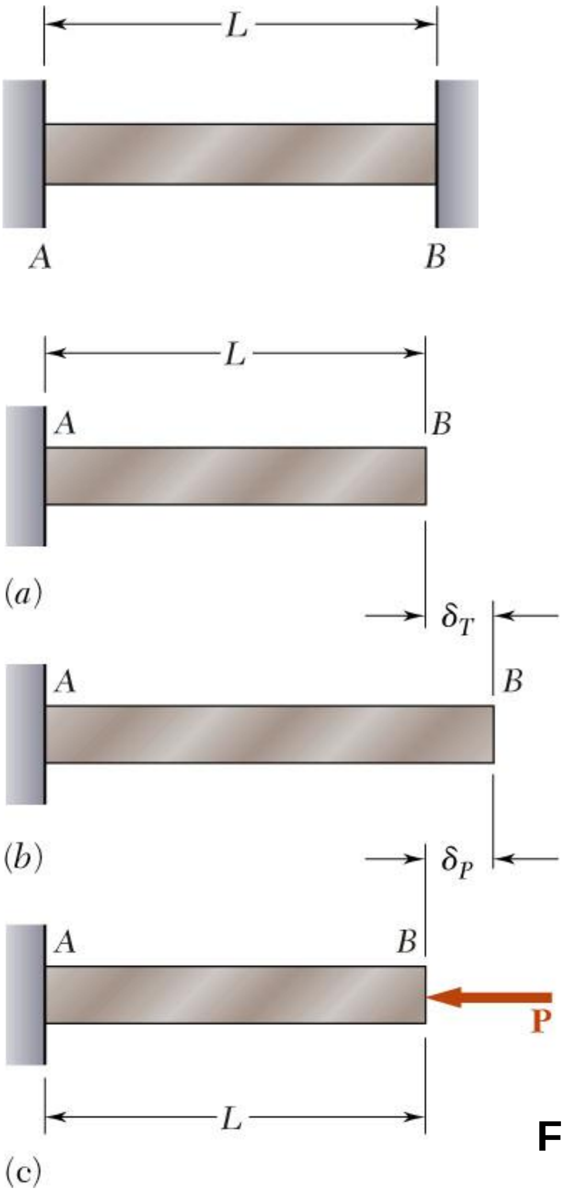


Fig. 2.27

- A temperature change results in a change in length or *thermal strain*. There is no stress associated with the thermal strain unless the elongation is restrained by the supports.

- Treat the additional support as redundant and apply the principle of superposition.

$$\delta_T = \alpha(\Delta T)L \qquad \delta_P = \frac{PL}{AE}$$

$\alpha =$ thermal expansion coef.

- The thermal deformation and the deformation from the redundant support must be compatible.

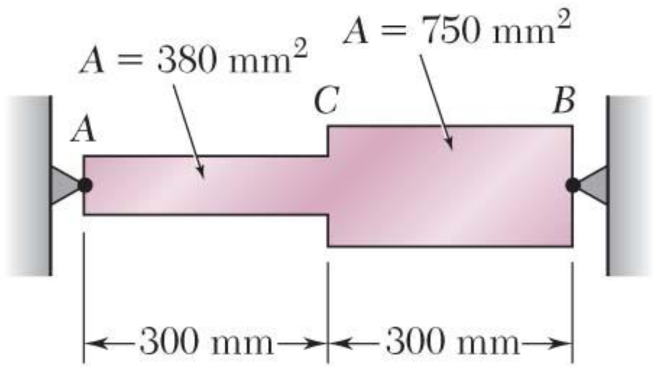
$$\delta = \delta_T + \delta_P = 0$$

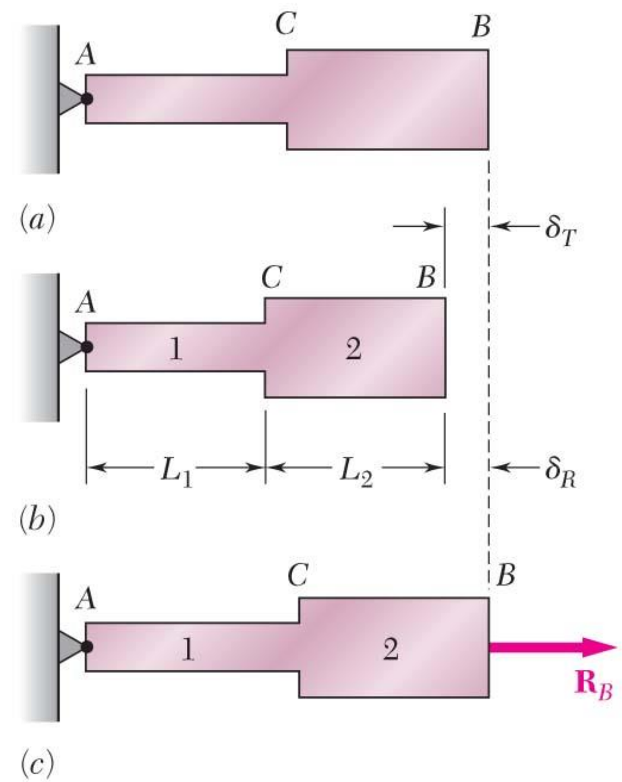
$$\alpha(\Delta T)L + \frac{PL}{AE} = 0$$

$$P = -AE\alpha(\Delta T)$$

$$\sigma = \frac{P}{A} = -E\alpha(\Delta T)$$

Concept Application 2.6



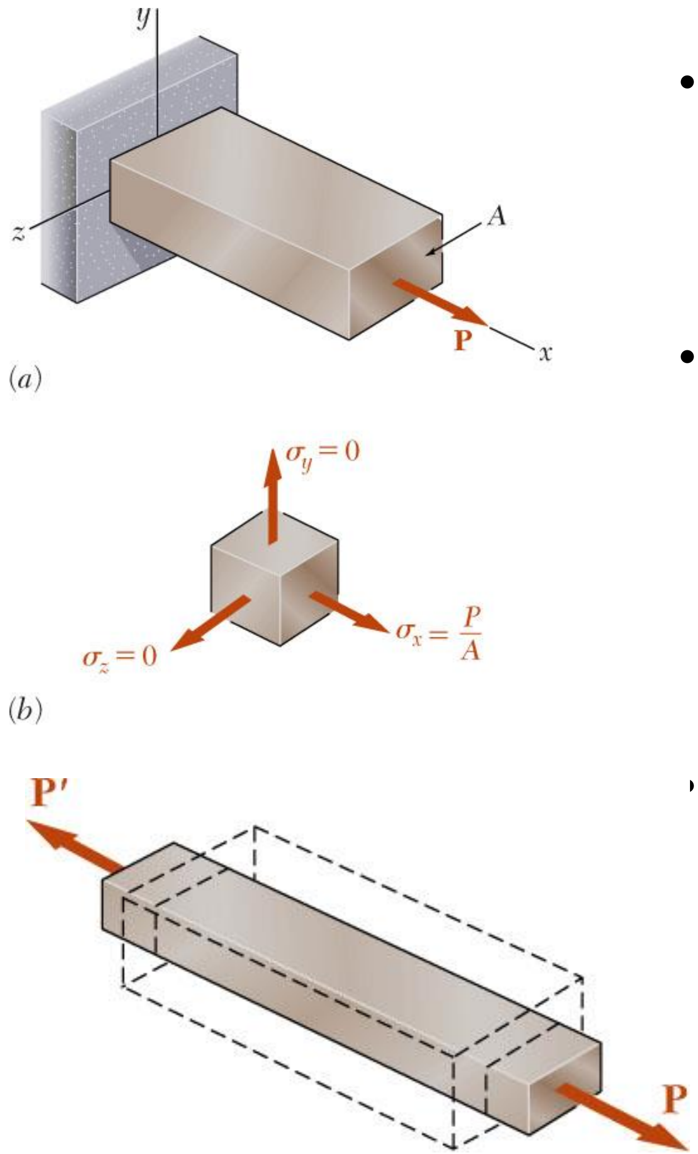


Concept Application 2.6

Problems

- Page 86 Sample problem 2.3
- Page 87 Sample problem 2.4

2.4 Poisson's Ratio



- For a slender bar subjected to axial loading:

$$\varepsilon_x = \frac{\sigma_x}{E} \qquad \sigma_y = \sigma_z = 0$$

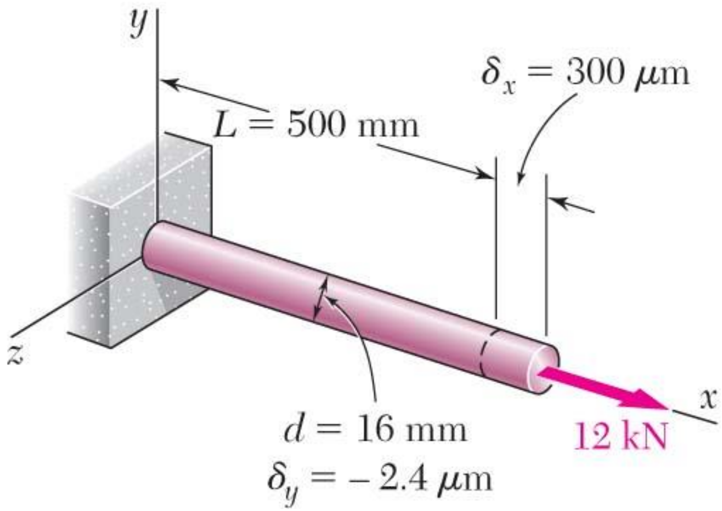
- The elongation in the x-direction is accompanied by a contraction in the other directions. Assuming that the material is isotropic (no directional dependence),

$$\varepsilon_y = \varepsilon_z \neq 0$$

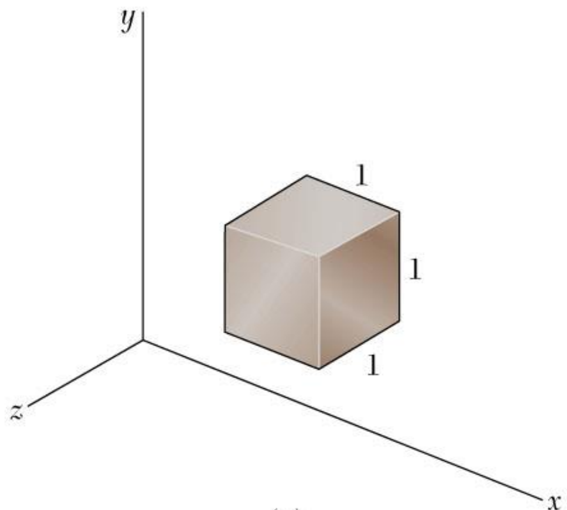
- Poisson's ratio is defined as

$$\nu = \left|\frac{\text{lateral strain}}{\text{axial strain}}\right| = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

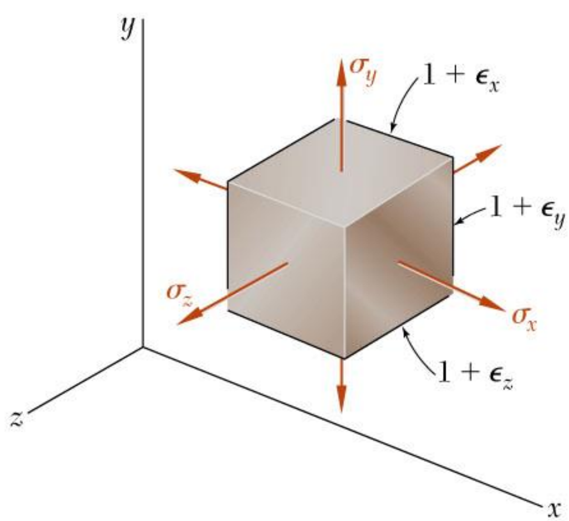
Concept Application 2.7



2.5 Multiaxial Loading: Generalized Hooke's Law



(a)



(b)

- For an element subjected to multi-axial loading, the normal strain components resulting from the stress components may be determined from the *principle of superposition*. This requires:

    1) strain is linearly related to stress
    2) deformations are small

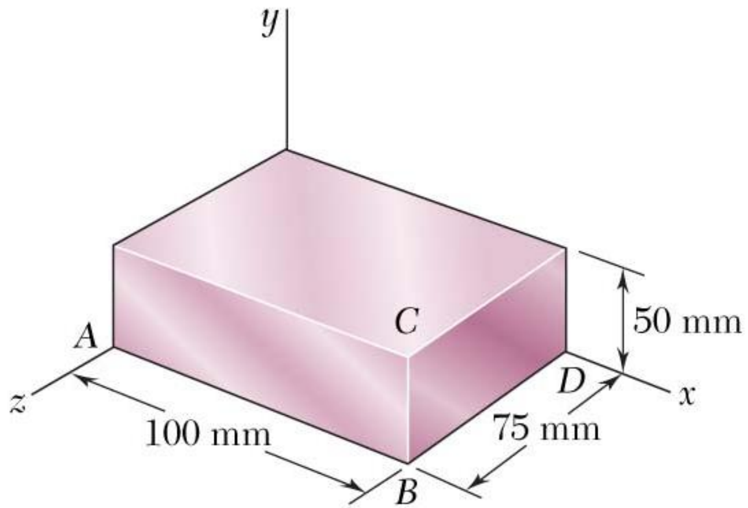
- With these restrictions:

$$\varepsilon_x = +\frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\varepsilon_y = -\frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\varepsilon_z = -\frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E} + \frac{\sigma_z}{E}$$

Concept Application 2.8 p97



2.7 Shearing Strain

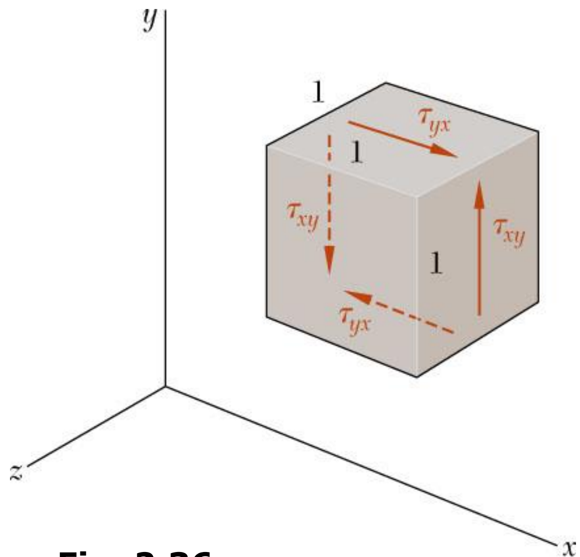


Fig. 2.36

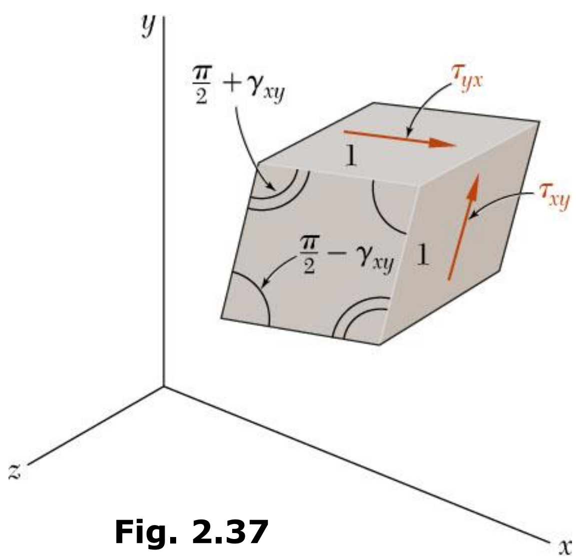


Fig. 2.37

- A cubic element subjected to a shear stress will deform into a rhomboid. The corresponding *shear* strain is quantified in terms of the change in angle between the sides,

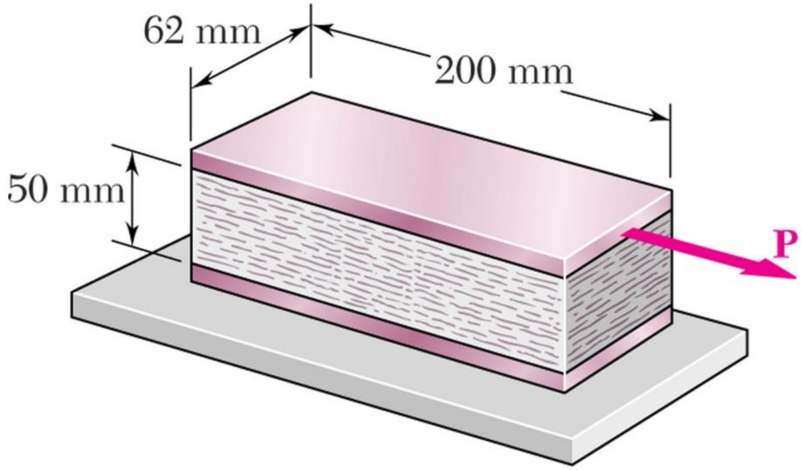
$$\tau_{xy} = f(\gamma_{xy})$$

- A plot of shear stress vs. shear strain is similar to the previous plots of normal stress vs. normal strain except that the strength values are approximately half. For small strains,

$$\tau_{xy} = G\gamma_{xy} \quad \tau_{yz} = G\gamma_{yz} \quad \tau_{zx} = G\gamma_{zx}$$

where *G* is the modulus of rigidity or shear modulus.

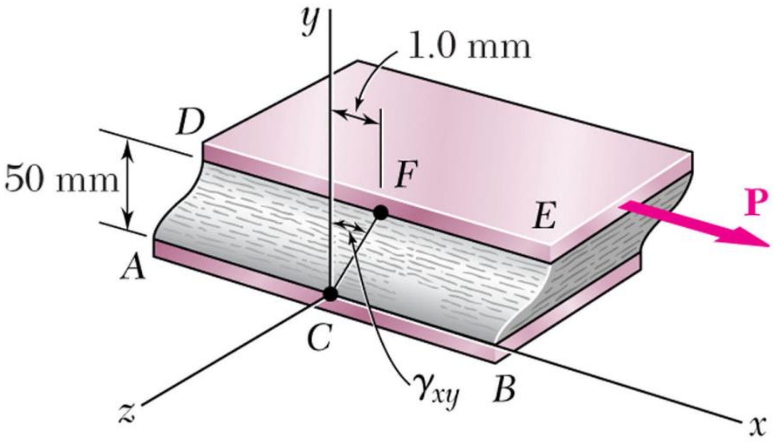
Concept Application 2.10 p102



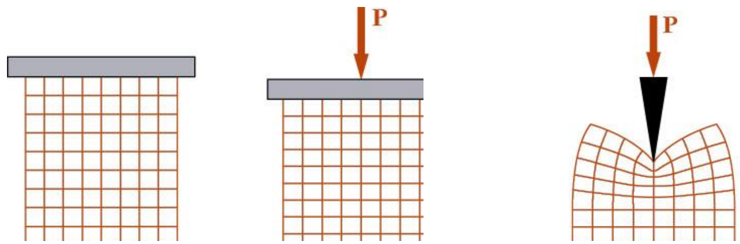
A rectangular block of material with modulus of rigidity G = 630 MPa is bonded to two rigid horizontal plates. The lower plate is fixed, while the upper plate is subjected to a horizontal force P. Knowing that the upper plate moves through 1.0 mm. under the action of the force, determine a) the average shearing strain in the material, and b) the force P exerted on the plate.

SOLUTION:

- Determine the average angular deformation or shearing strain of the block.
- Apply Hooke's law for shearing stress and strain to find the corresponding shearing stress.
- Use the definition of shearing stress to find the force P.

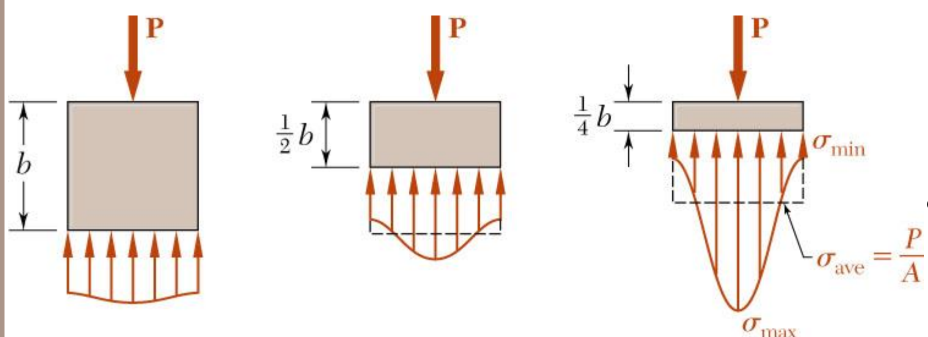

y
1.0 mm
D
50 mm
F
E
P
A
C
γ_{xy}
B
z
x

2.10 Stress and Strain Distribution under Axial Loading: Saint-Venant’s Principle p115



- Loads transmitted through rigid plates result in uniform distribution of stress and strain.

- 聖維南原理（Saint Venant’s Principle）是彈性力學的基礎性原理，是法國力學家聖維南於1855年提出的。其內容是：分佈於彈性體上一小塊面積（或體積）內的荷載所引起的物體中的應力，在離荷載作用區稍遠的地方，基本上只同荷載的合力和合力矩有關；荷載的具體分佈只影響荷載作用區附近的應力分佈。



$\sigma_{min} = 0.973\sigma_{ave}$
$\sigma_{max} = 1.027\sigma_{ave}$

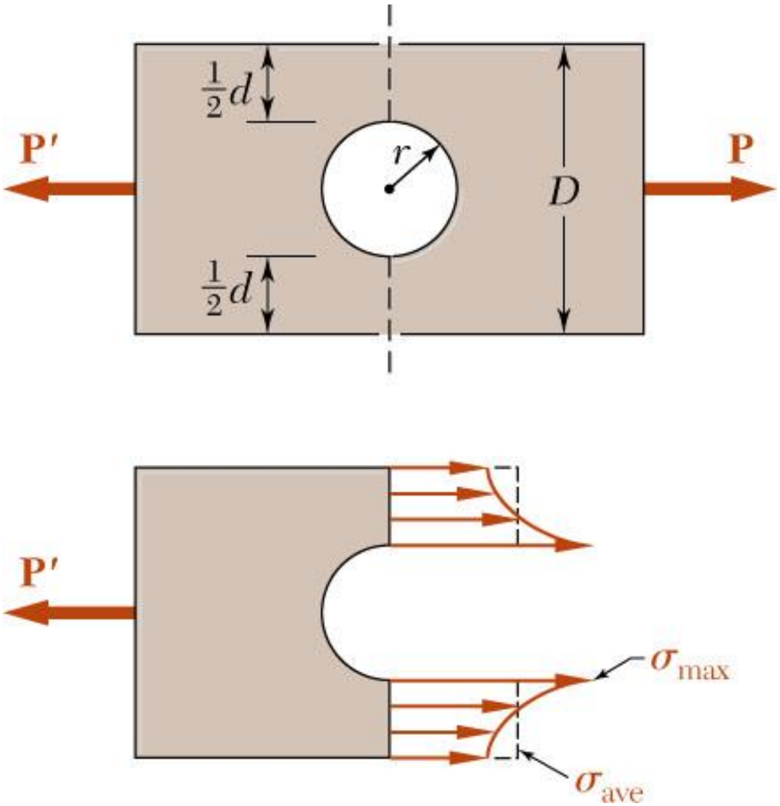
$\sigma_{min} = 0.668\sigma_{ave}$
$\sigma_{max} = 1.387\sigma_{ave}$

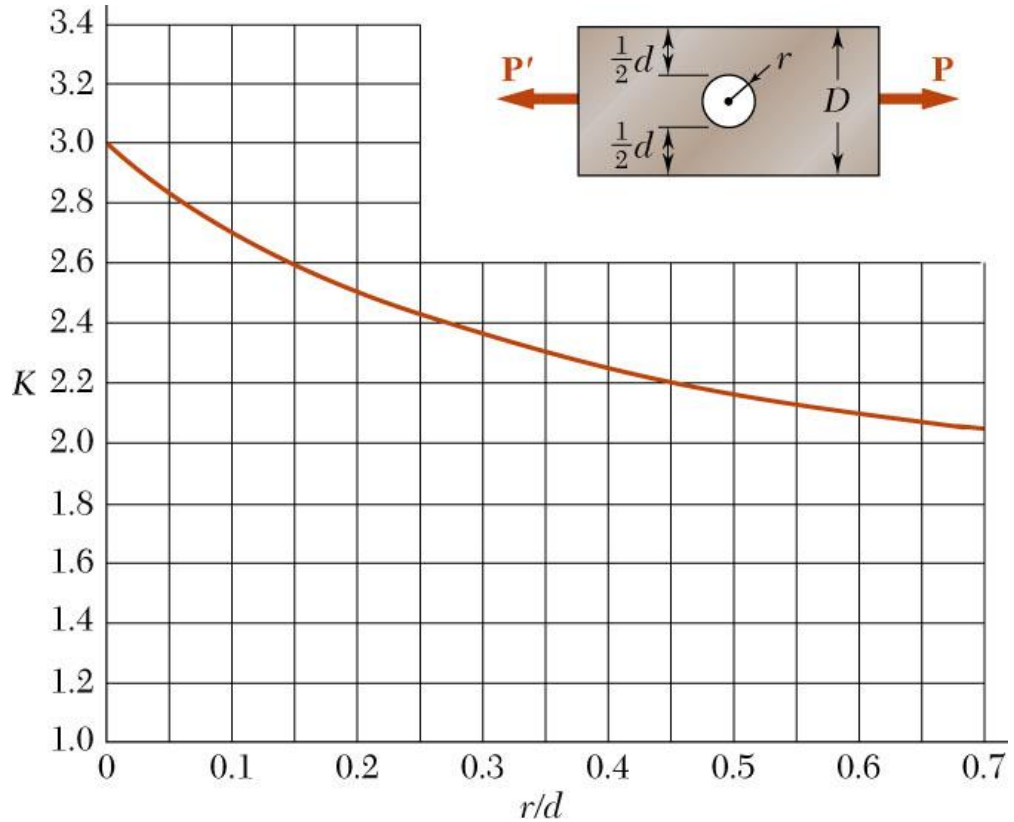
$\sigma_{min} = 0.198\sigma_{ave}$
$\sigma_{max} = 2.575\sigma_{ave}$

- Stress and strain distributions become uniform at a relatively short distance from the load application points.

- **Saint-Venant’s Principle:**
  Stress distribution may be assumed independent of the mode of load application except in the immediate vicinity of load application points.

2.11 Stress Concentrations: Hole



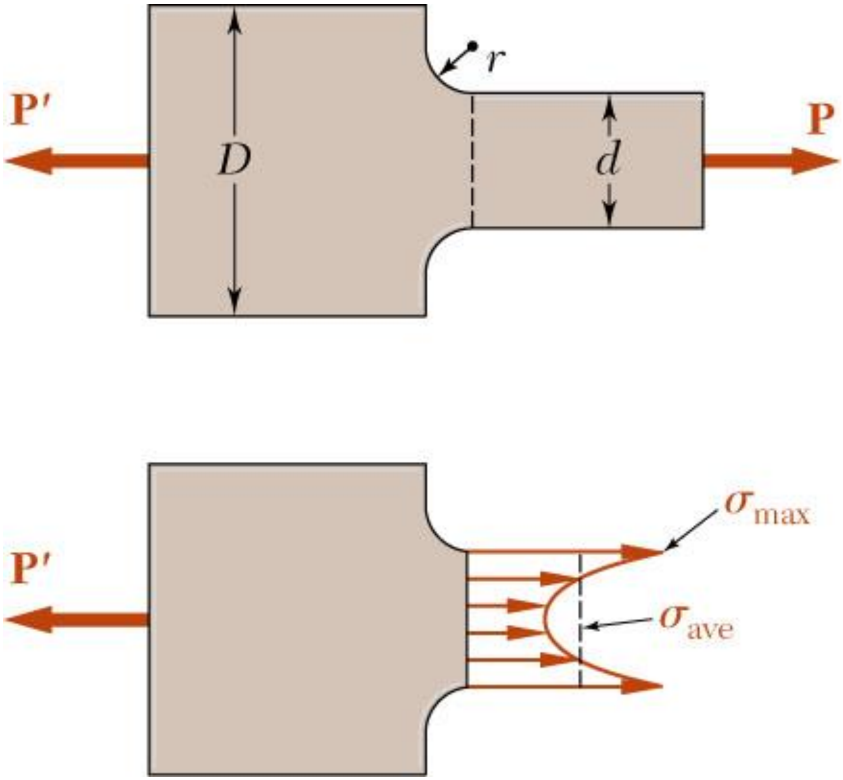


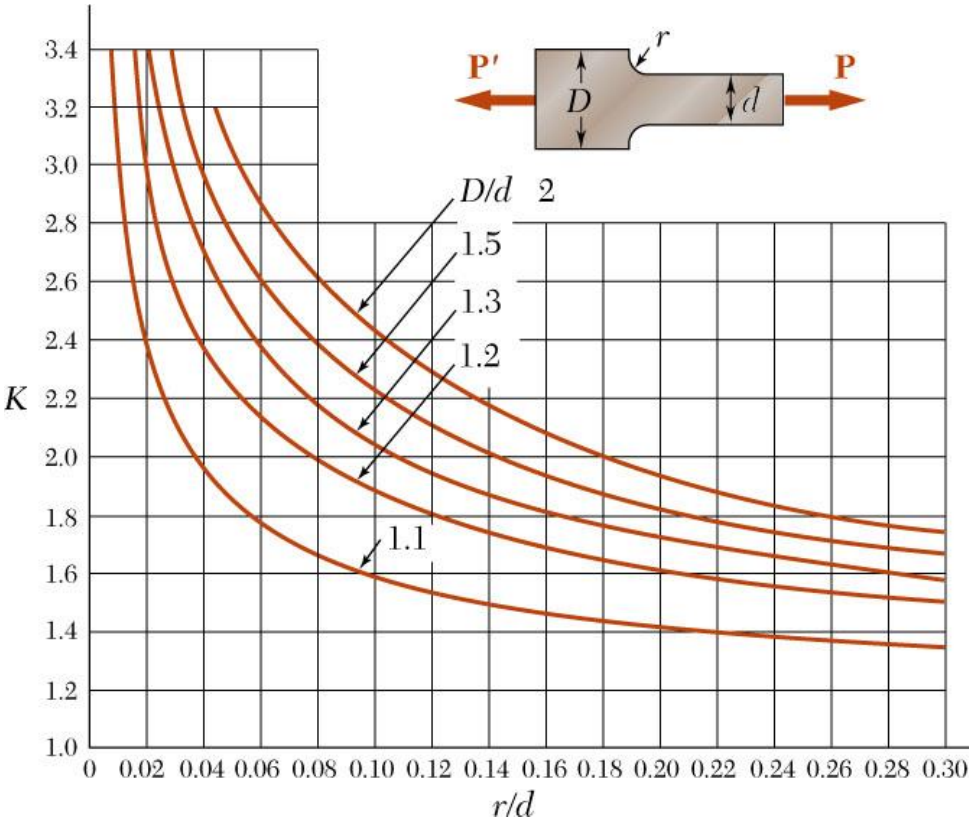
***(a)* Flat bars with holes**

Discontinuities of cross section may result in high localized or *concentrated* stresses.

$$K = \frac{\sigma_{max}}{\sigma_{ave}}$$

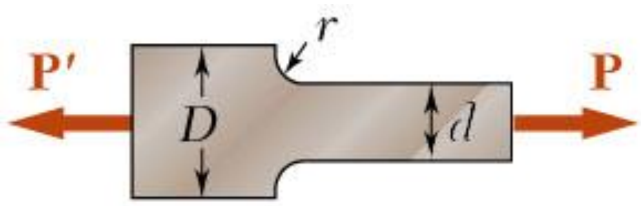
Stress Concentration: Fillet





***(b)* Flat bars with fillets**

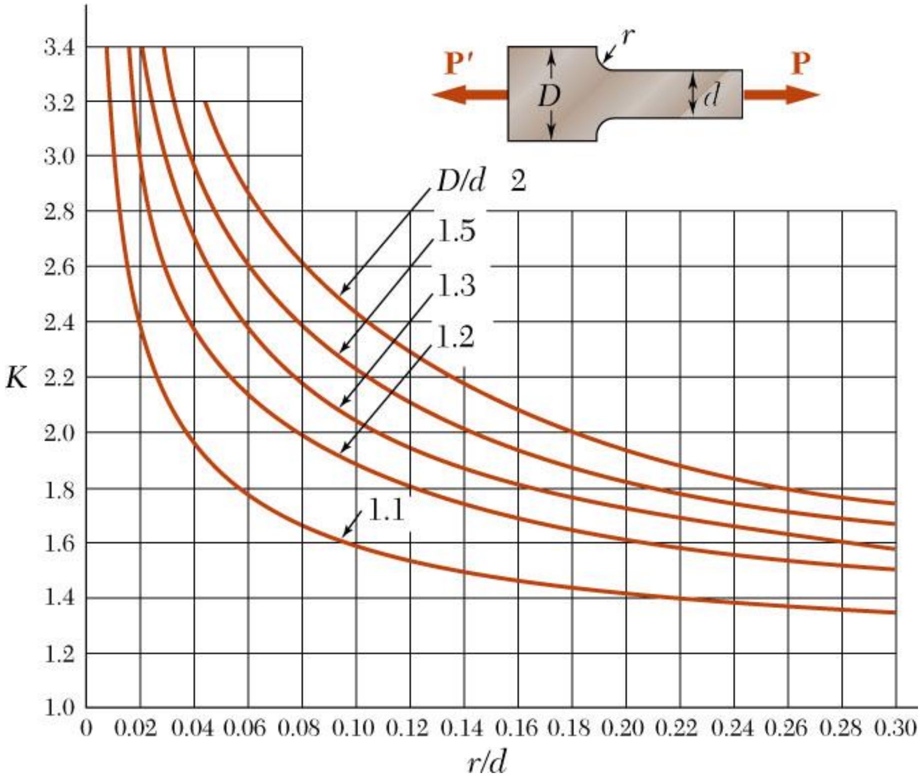
Concept Application 2.12



Determine the largest axial load *P* that can be safely supported by a flat steel bar consisting of two portions, both 10 mm thick, and respectively 40 and 60 mm wide, connected by fillets of radius *r* = 8 mm. Assume an allowable normal stress of 165 MPa.

SOLUTION:

- Determine the geometric ratios and find the stress concentration factor from Fig
- Find the allowable average normal stress using the material allowable normal stress and the stress concentration factor.
- Apply the definition of normal stress to find the allowable load.



(b) **Flat bars with fillets**

Problems

- Page 131 2.113
- Page 141 2.127